# SYNCHRONIZATION OF PENDULA ROTATING IN DIFFERENT DIRECTIONS

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*Summary* We study the synchronization of a number of pendula mounted on a horizontal beam which can roll on the parallel surface. Under the driving moment, the pendula rotate in different directions: one of them rotates counterclockwise, the rest rotate clockwise. It has been shown that after a transient different types of phase synchronization between pendula can be observed, despite opposite directions of rotations.

## INTRODUCTION AND MOTIVATION

Mechanical systems that contain rotating parts (for example vibro-exciters, unbalance rotors) are typical in engineering applications and for years have been the subject of intensive studies [1]. One problem of scientific interest, which among others occurs in such systems is the phenomenon of synchronization of different rotating parts [2,3 and references within]. Despite different initial conditions, after a sufficiently long transient, the rotating parts move in the same way - complete synchronization, or a permanent constant shift is established between their displacements, i.e., the angles of rotation - phase synchronization [2,4]. Synchronization occurs due to dependence of the periods of rotating elements motion and the displacement of the base on which these elements are mounted.

In the previous paper [4] we consider the dynamics of the system consisting of $n$ pendula mounted on the movable beam. The pendula are excited by the external torques which are inversely proportional to the angular velocities of the pendula. As the result of such excitation each pendulum rotates around its axis of rotation. It has been assumed that all pendula rotate in the same direction. We consider the case of slowly rotating pendulums and consider the influence of the gravity on their motion. It has been shown that both complete and phase synchronizations of the rotating pendula are possible. We derive the approximate analytical conditions for both types of synchronizations and equations which allow the estimation of the phase differences between the pendula. Contrary to the case of oscillatory pendulums [5,6] phase synchronization is not limited to three and five clusters configurations. Our results have been compared to these of [2].

In this paper we study the dynamics of the similar system as in [4] but this time we assume that one pendulum (numbered 1) rotates counterclockwise, i.e., has a positive angular velocity angular while the remaining pendula rotates clockwise with negative angular velocity. We consider two cases: (i) pendula rotate in the horizontal plane, i.e., the gravity has no influence on their motion, (ii) pendula rotate in the vertical plane and the weight of them causes the unevenness of their rotation, i.e., the pendula slow down when the center of its mass goes up and accelerates when the center of its mass goes down. We show that in such systems, despite opposite directions of rotation different types of synchronization occur. We give evidence that our results are robust as they exist in the wide range of system parameters.

## MODEL OF THE SYSTEM WITH HEAVY ELASTIC ELEMENTS AND APPLIED METHODS

We consider the system shown in Figure 1. It consists of a rigid beam of mass $M$ on which $n$ identical rotating pendula are mounted. The beam is connected to a stationary base by the spring with a stiffness coefficient $k_x$ and a damper with a damping coefficient $c_x$. Due to the existence of the forces of inertia, which act on each pendulum pivot, the beam can move in the horizontal direction (this motion is described by coordinate $x$). The masses of the pendula are indicated as $m$. $B$ is the moment of inertia with respect to the axis of rotation. $l$ is the distance from the axis of rotation to the center of the pendulum's mass. Rotation of the $i$-th pendula is described by $\varphi_i$. The rotations of the pendula are damped by linear dampers (not shown in Figure 1) with damping coefficient $c_\varphi$. Each pendulum is driven by the drive moment inversely proportional to their velocity: $N_{0i} - \dot{\varphi}_i N_1$. If any other external forces do not act on the pendulum, then under the action of such a moment it rotates with constant angular velocity. If the system is in a gravitational field ($g$=9.81[m/s$^2$] - acceleration of gravity), the weight of the pendulum causes the unevenness of its rotation: the pendulum slows down, when the center of mass rises up and accelerates when the center of mass falls down. It is assumed that $N_1$>0.0. If $N_{0i}$ torque is positive, the pendulum rotates to the left having a positive value of the instantaneous angular velocity, if $N_{0i}$<0.0, the pendulum rotates to the right with a negative angular velocity.

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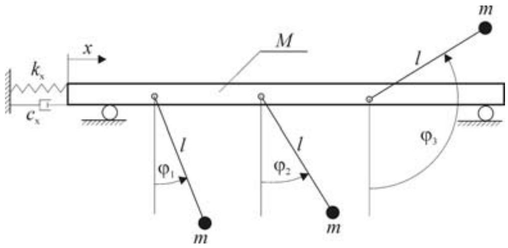

Figure 1: Pendula suspended on the movable beam.

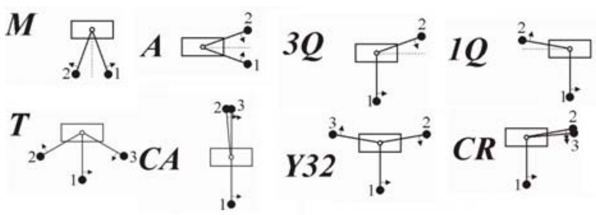


Figure 2. Pendula’s configurations

The equations of motion described above are as follows:

$$B\,\ddot{\varphi}_i + m\ddot{x}l\cos\varphi_i + c_\varphi\dot{\varphi}_i + mgl\sin\varphi_i = N_{0i} - \dot{\varphi}_i N_1, \tag{1}$$

$$(M + nm)\ddot{x} + c_x\dot{x} + k_x x = \sum_{i=1}^{n} m\,l\left(-\ddot{\varphi}_i\cos\varphi_i + \dot{\varphi}_i^2\sin\varphi_i\right), \tag{2}$$

where $i$=1,2,…$n$. In our numerical simulations eqs.(1,2) have been integrated by the 4th order Runge-Kutta method. The obtained results confirmed the existence of the phenomenon of phase synchronization in the considered system and allowed the determination of phase angles between the synchronized pendula. Additionally the numerical integration of eqs.(1,2) allows the determination of the basins of attraction of different coexisting configurations of the synchronized pendula. We use the following parameters' values: $m$=1.00 [kg], $l$=0.25 [m], $c_\varphi$=0.01 [Nsm], $N_1$=0.50 [Nsm], $M$=6.00 [kg]. It has been considered that pendulum 1 rotates counterclockwise and pendula 2,…$n$ clockwise so $N_{01}$=5.00 [Nm], $N_{0i}$=-5.00 [Nm] ($i$=2,…,$n$). We assume that at the initial state: $x_0$=0.0, $\dot{x}_0 = 0$ (the beam is at rest) and the pendula are rotating in opposite directions with equal velocities $\dot{\varphi}_{10} = \omega_1 = 10.0$ [s$^{-1}$], $\dot{\varphi}_{i0} = \omega_i = -10.0$ [s$^{-1}$], $i$=2,…,$n$. Other initial conditions are given by the pendula’s initial positions $\varphi_{i0}$.

## CONCLUSIONS

In the system in which the pendula rotate in opposite directions one can observe different types of synchronization. During the synchronous motion the average (over the period of rotation) value of the sum of angular displacements of pendula rotating to the right and left is constant. Our approximate analytical studies of synchronization momenta, i.e., momenta responsible for energy transfer between pendula, allow prediction of the mirror (*M*), antiphase (*A*) synchronization in the system of $n$=2 pendula and tree (*T*) and cluster antiphase (*CA*) synchronization in the system of $n$=3 pendula (see Figure 2). These types of synchronization occur for both horizontal (without gravity) and vertical (in the gravitational field) planes of pendula rotation.

Numerical simulations confirm the existence of these types of synchronization. In the system with two pendula they additionally show that due to gravity, at sufficiently large values of the stiffness coefficient of stiffness $k_x$, one can observe two modifications of the (*A*) type synchronization, namely the first-quarter (*1Q*) and the third-quarter (*3Q*) synchronization. In the system with three pendula we show that the analytically predicted tree (*T*) and cluster antiphase (*CA*) types of synchronization occur independently of the consideration of gravity. For pendula rotation in the gravitational field one can also observe, for sufficiently large values of the stiffness coefficient $k_x$, two modifications of the (*T*) type, i.e., yankee (*Y23*) and yankee (*Y32*) synchronization and two modifications (*CA*) type: (*CL*) and (*CR*) synchronization. Typical pendula’s configurations are shown in Figure 2.

The types of synchronous configurations identified for the system of three pendula can be observed in the systems with larger number of pendula. Contrary to the case of oscillating pendulums [5,6] rotating pendula are not grouped in three or five clusters only. The lack of this restriction causes that in the system (1,2) depending on initial condition one can observe a great variety of different synchronization configurations. The number of configurations grows with a number of pendulums $n$.

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