RECTILINEAR MOTIONS OF MULTI-MODULE VIBRATION-DRIVEN SYSTEMS: EFFECTS OF THE PHASE SHIFTS AMONG INTERNAL MOTIONS

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Summary In this paper, rectilinear motions of multi-module vibration-driven systems on resistant media are developed. Major attention is focused on the effects of the phase shifts among internal masses on the rectilinear motions of the system as a whole. For both two-module and three-module systems, by controlling the phase shifts, better performance of the systems's motions with higher average steady-state velocities is realized. In view of the non-smooth factor in Coulomb's dry friction model, mechanisms for the possible stick-slip motions, which closely relate to the phase shifts among the internal motions, are put forward.

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INTRODUCTION

Multi-module vibration-driven systems, which are simple in design and whose bodies can be fabricated into small size, can be utilized as the dynamic models of worm-like robots. Such modeling is the developing trend of robot research.

System with movable internal masses is a typical vibration-driven system. F.L. Chernousko is the precursor for studying the rectilinear motion of a rigid body with a controlled movable internal mass along a rough horizontal plane. Optimal control was mainly concerned in[1]. Later, rectilinear motions of chain of bodies connected to one another by means of some types of mechanical components have been studied by Zimmermann, Ziedis and Bolotnik, et al. Work [2] dealt with a system composed of two identical modules with unbalanced vibration exciters. The undulatory motion of a straight chain of three equal particles interconnected with kinematical constraints was considered in[3].

This paper describes theoretical and numerical efforts concerning the effects of the phase shifts among internal masses on the rectilinear motions of multi-module vibration-driven systems. A two-module and a three-module system are considered, respectively. Mechanisms for possible stick-slip motions that closely relate to the phase shifts are also presented.

OPTIMIZATION OF THE RECTILINEAR MOTIONS

Optimization of the rectilinear motions of a two-module and a three-module vibration-driven system are presented in this section. Each module of the studied systems is composed of a rigid body and a controlled movable internal mass. The phase shifts among the internal masses are taken as the optimal object, and improving the performance of motion is set as the optimization purpose. For the two-module vibration-driven system shown in Figure 1, an acceleration-controlled mode is adopted by the internal mass, and an anisotropic linear resistance is assumed to act between the system and the environment. For the three-module system, we assume that the internal motions have the sinusoidal form and the resistance force obey anisotropic Coulomb's dry friction law, as is shown in Figure 2.

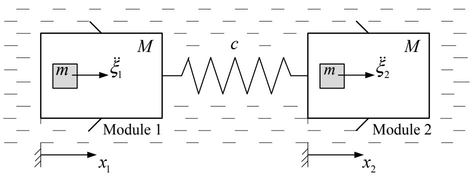

Figure 1. Model of a two-module system

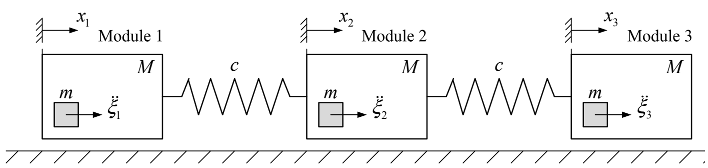

Figure 2. Model of a three-module system

Two-module vibration-driven system

In the two-module system, both the two internal masses vibrate with the same frequency and with the same amplitude, while are shifted in phase τ_0 ($\tau_0 \in [0,\, T]$). Primary resonance situation that the excitation frequency ω is close to the

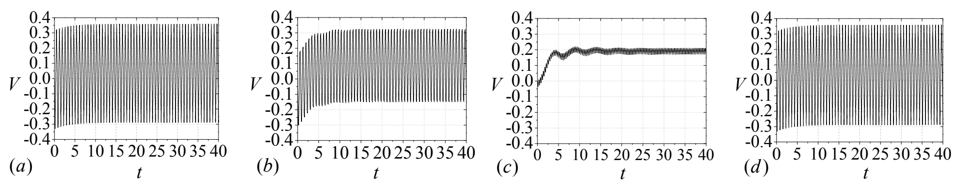

Figure 3. Time histories of the steady-state velocity of the two-module system: (a)$\tau_0 = 0$s, (b)$\tau_0 = 0.1$s, (c)$\tau_0 = 0.35$s,(d)$\tau_0 = 0.55$s

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frequency of natural elastic oscillation ω_0 is mainly discussed. Due to the existence of non-smooth factor in the external resistance force, method of averaging is employed to obtain the steady-state motion of the system. Theoretical analysis shows that the average steady-state velocity can be controlled by varying the phase shifts between the two internal motions. An optimal value of the phase shift is obtained, which does not vary with the change of external environments.

Numerical simulations are carried out to verify the theoretical results. For a set of given parameters, the optimal situation occurs when $\tau_0 = 0.35$s. From Figure 3, it is clearly that when $\tau_0 = 0.35$, the average steady-state velocity reaches maximum, accompanying with the smallest velocity amplitude.

Three-module vibration-driven system

Similarly, in the three-module system, the three internal masses are assume to vibrate with the same frequency and with the same amplitude, but are shifted in phase. Due to the periodicity of the internal controls, it is the phase shifts between every two internal masses that affect the steady-state motion of the system. Without loss of generality, the phase of the middle internal mass is set as zero, i.e., $\phi_{20} = 0$, and the phases of the other two internal masses are alterable.

The study shows that by modifying the phase shifts among the internal motions, one can increase the average steady-state velocity of the system as a whole. Hence, an optimal control strategy is put forward that when $|\phi_{10} - \phi_{30}| \approx \pi$, a maximal average steady-state velocity of the system can be obtained, without extra energy input. Results of numerical simulations for both the uncontrolled and optimal-controlled situations are revealed in Figure 4. One can clearly observe that without extra energy input, better performance of the motion with a higher average steady-state velocity is realized.

What worth mention is that the above optimization relating to the phase shift may also involves synchronization issue of the modules in a system, which requires further investigation.

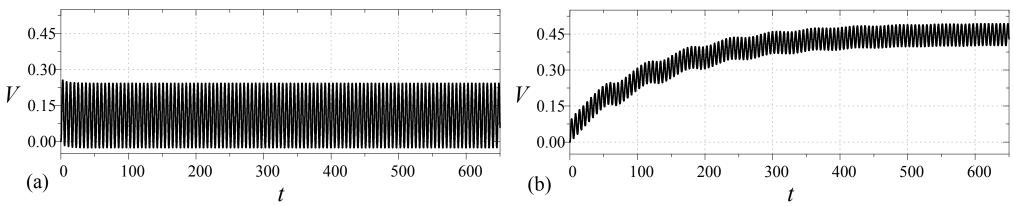

Figure 4. Time histories of the steady-state velocity V of the three-module system for the uncontrolled situation and optimal-controlled situation when $k = 0.25$: (a) uncontrolled situation $\phi_{10} = \phi_{20} = \phi_{30}$, (b) optimal-controlled situation $\phi_{10} = 0$, $\phi_{20} = 0$, $\phi_{30} = 3.1$.

MECHANISMS FOR STICK-SLIP MOTIONS

In the three-module vibration-driven system, stick-slip motions may occur on each module, which is a typical phenomenon of non-smooth dynamical system containing Coulomb's dry friction and cannot be ignored. Recent work showed that the stick-slip effect can even be utilized to further optimize a single-module vibration-driven system[4]. In this research, two mechanisms for these possible stick-slip motions are put forward. It is shown that both the two mechanisms have close relation with the phase shifts among the internal motions. It is these stick-slip motions that prevent us from obtaining the steady-state motions of the system directly by method of averaging. Through several numerical examples, the theoretical mechanisms are verified and the predicted stick-slip phenomena are observed.

CONCLUSION

Effects of the phase shifts among internal masses on the rectilinear motions of multi-module vibration driven systems are studied in this paper. It is shown that the phase shifts play a key role in the optimization process, with the purpose of improving the performance of systems' rectilinear motions. Mechanisms for possible stick-slip motions in a three-module system are also discussed in this paper.

Future work includes the study on steady-state motions of N-module (N>3) vibration-driven systems and their optimal control strategies, both theoretically and experimentally. The mechanisms and the possible applications of stick-slip effects in such systems are also worth to be studied.

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