VORTEX-INDUCED VIBRATIONS OF PIPES CONVEYING FLUID IN SUBCRITICAL AND SUPERCRITICAL REGIMES

Huliang Dai & Lin Wang[a)]
Department of Mechanics, Huazhong University of Science and Technology, Wuhan, 430074, China
Hubei Key Laboratory for Engineering Structural Analysis and Safety Assessment, Wuhan, 430074, China

Summary In this paper the effects of both subcritical and supercritical internal fluid flows on the dynamical behaviours of pipes subjected to vortex-induced vibration are analyzed. When the structural natural frequency is close to the vortex-shedding frequency, it is found that the pipe would display periodic motions for subcritical internal fluid velocity, and that for supercritical internal fluid velocity, however, complex dynamical behaviours including period-doubling bifurcations and chaotic motion could occur.

INTRODUCTION

Vortex-induced vibration (VIV) commonly occurs in ocean engineering when the pipe is subjected to lateral sea currents. Therefore, the problem of pipes undergoing VIV is one of the hot topics across applied mechanics. Most of the early studies on VIV of pipes are restricted only to pipes without internal fluid flow [1,2]. In recent years, a series of theoretical and experimental studies on the vibration characteristics of pipes with axial internal fluid flow subjected to VIV were reported by Guo et al. and Keber and Wiecigroch (see, e.g., [3,4]). In these previous studies, however, the velocities of internal axial fluid flow were chosen to be lower than the critical value. In the current paper, of particular interest is to analyze the VIV of fluid-conveying pipes with subcritical and supercritical internal fluid velocities.

PROBLEM FORMULATION

The system under consideration consists of a slender pipe conveying internal axial fluid in a uniform cross-flow, as shown in Fig. 1. The pipe is simply supported at both ends. It is assumed that the motions of the flexible pipe will be planar perpendicular to the external flow.

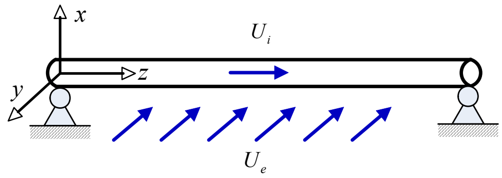

Fig.1. Schematic of a simply supported pipe subjected to VIV

If gravity, internal damping, externally imposed tension and pressurization effects are either absent or neglected, the equation of planar motion for the fluid-conveying pipe subjected to a sheared flow may take the following form

$$m\frac{\partial^2 x(z,t)}{\partial t^2}+EI\frac{\partial^4 x(z,t)}{\partial z^4}+2m_f U_i\frac{\partial^2 x(z,t)}{\partial z\partial t}+m_f U_i^{\,2}\frac{\partial^2 x(z,t)}{\partial z^2}-\frac{EA_p}{2L}\left[\int_0^L\left(\frac{\partial x(z,t)}{\partial z}\right)^2 dz\right]\frac{\partial^2 x(z,t)}{\partial z^2}=f(z,t) \qquad (1)$$

where $m=m_f+m_p+m_d$, m_f, m_p and m_d are respectively defined as the mass per unit length of the internal fluid, the pipe and the added fluid, EI is the flexural rigidity of the pipe, A_p is the cross-section area of the pipe, L is the pipe length, U_i is the velocity of the internal flow; $f(z,t)$ represents the vortex-induced force acting on the pipe; $x(z,t)$ is the lateral displacement of the pipe, z and t are the axial coordinate and time, respectively. The force $f(z,t)$ may be given by

$$f(z,t)=f_D(z,t)+f_L(z,t)=-\frac{1}{2}C_D\rho_o DU_e\frac{\partial x(z,t)}{\partial t}+\frac{1}{4}C_{L_0}\rho_o DU_e^{\,2}q(z,t) \qquad (2)$$

where $f_D(z,t)$ is the drag force acting in the lateral direction, $f_L(z,t)$ is the lift force; U_e is the external flow velocity, ρ_o is the density of the external fluid, C_D=1.2 and C_{L0}=0.3 are the coefficients of drag and lift forces in the transverse direction; D is the outside diameter of the pipe,. In Eq. (2), $q(z,t)$ is defined as the reduced lift coefficient, which describes the behavior of the vortex-induced wake and may be modeled by the following van der Pol equation [4]

$$\frac{\partial^2 q}{\partial t^2}+\delta\omega_s(q^2-1)\frac{\partial q}{\partial t}+\omega_s^{\,2}q=\frac{P}{D}\frac{\partial^2 x}{\partial t^2} \qquad (3)$$

where ω_s is the vortex-shedding frequency, the values of δ and P are constant and may be obtained through experiment. We may choose δ = 0.3 and P = 12 [4]. The infinite analytical model described by Eqs. (1) and (3) will be discretized by using a two-mode Galerkin approximation in the following analysis.

RESULTS

For simply supported fluid-conveying pipes without external cross-flow, divergence (buckling) is the expected form of instability. The internal fluid velocity at which the pipe would lose stability is usually defined as the critical fluid velocity. Since the dynamical behaviours of fluid-conveying pipes in subcritical and supercritical regimes are

[a)] Corresponding author. Email: wanglindds@mail.hust.edu.cn.

fundamentally different, they will be treated separately. In the following calculations, the Young's modulus E=210GPa, the length L=150m, the outer diameter D=0.25m, the inner diameter D_i=0.125m, the density of the pipe ρ_p=7850kg/m^3, the density of the external fluid ρ_o=1020kg/m^3 and the density of the internal fluid ρ_i=870 kg/m^3.

VIV of pipes conveying fluid in subcritical regime

In the subcritical regime (the internal fluid velocity is lower than the critical value), the vibration of the pipe is always around the original equilibrium. In this case, Eqs. (1) and (3) after Galerkin discretization to two d.o.f. may be analyzed by using a method of multiple scales. We will focus on the dynamics of pipe under the lock-in conditions, i.e., $\Omega_s=\Omega_1+\varepsilon\sigma$, where $\Omega_s=\omega_s[m/(EI)]^{1/2}L^2$ is the dimensionless vortex-shedding frequency, Ω_1 is the dimensionless natural frequency of the pipe, ε and σ are small perturbation parameter and detuning parameter, respectively.

The steady-state responses of the pipe in subcritical regime are shown in Fig. 2. It is observed that the amplitude of the periodic motion decreases with increasing dimensionless internal fluid velocity ($v=[m_f/(EI)]^{1/2}LU_i$) for a given σ. The lock-in region plotted in the (σ-A/D) plane becomes narrower with increasing v.

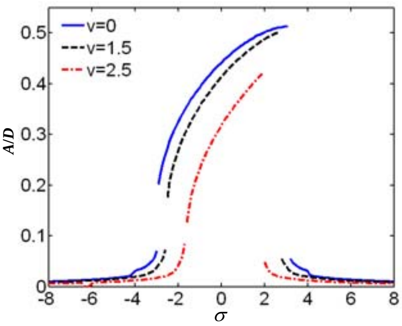

Fig.2. Steady-state response in subcritical regime

VIV of pipes conveying fluid in supercritical regime

In the supercritical regime (the internal fluid velocity is higher than the critical value), the vibration of the pipe is always around the non-trivial equilibrium. In such a case, the dynamic displacement $x(z,t)$ may be assumed to be $x(z,t)=\psi(z,t)+x_0(z)$, where $x_0(z)$ represents the buckled configuration of the pipe without external cross-flow. With the aid of a fourth-order Runge-Kutta integration algorithm, typical numerical results for dynamical behaviours in supercritical regime are illustrated in Figs. 3 and 4. Fig. 3 displays a bifurcation diagram in which the internal fluid velocity was chosen to be the variable parameter. It is obviously seen that the pipe may undergo complex motions. A period-doubling bifurcation has been detected in the calculations. Sample results are also represented in the form of phase portraits (see Fig. 4). It is found that both periodic and chaotic motions would occur in the supercritical regime.

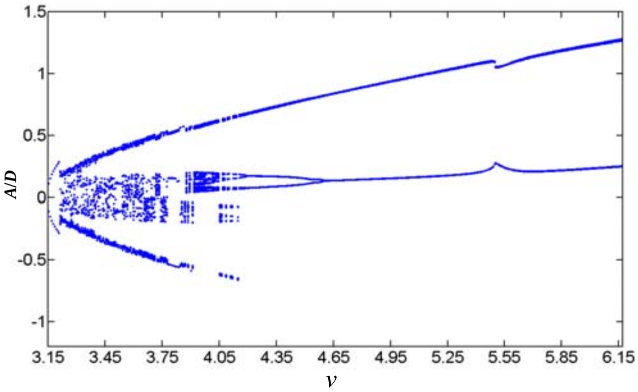

Fig.3. Bifurcation diagram for a system in supercritical regime

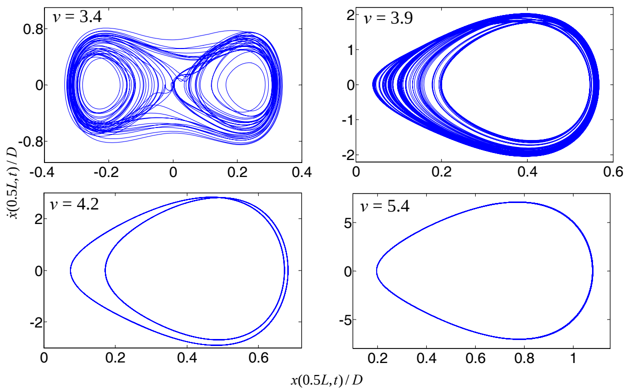

Fig.4. Periodic and chaotic motions for a system in supercritical regime

CONCLUSIONS

In this paper the dynamical behaviours of fluid-conveying pipes subjected to vortex-induced vibration are analyzed. Both subcritical and supercritical internal fluid flows have been considered. When the structural natural frequency is close to the vortex-shedding frequency, results showed that the pipe would display periodic motions for subcritical internal fluid velocity, and that for supercritical internal fluid velocity, however, both periodic and chaotic motions could occur.

Acknowledgements

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