MODAL INTERACTION IN THE INEXTENSIONAL BEAMS ON THE ELASTIC FOUNDATIONS

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Summary In this study, two-mode nonlinear response of the inextensional beam resting on the elastic foundation is investigated. The multimode discretization and method of multiple scales are applied to obtain the modulation equations, and the primary resonance excitations are considered. The equilibrium and dynamic solutions of modulation equations are examined, and the numerical simulations are performed to investigate the chaotic dynamics of the beam.

INTRODUCTION

Some soil-structure interaction problems can be approximately modeled as the beam resting on the elastic foundation. Many studies have been concerned with the linear vibrations of the beam on the elastic foundation. However, modern engineering design of the beam should consider the knowledge of the nonlinear vibration, which may be excited by the earthquake excitation or machine oscillation. Recently, the nonlinear response of the beam on the elastic foundation have been investigated [1]. However, a variety of unexpected nonlinear dynamics phenomena, due to the nonlinear modal interaction, have not been effectively revealed.

For the continuous systems with the internal resonances, due to the conservative character of the system, the effective nonlinear coefficients exhibit the symmetry and resonance characters [2, 3]. However, some nonlinearities of the beam on the elastic foundation may come from the second-order moment of the subgrade reaction [1]. The conservative character and these conditions may be disrupted. Therefore, more complex nonlinear dynamics of the beam on the elastic foundation would be expected.

In this study, the nonlinear interaction of the beam on the elastic foundation, due to the two-to-one internal resonance ($\omega_2 \approx 2\omega_1$), is investigated. The method of multiple scales is applied to obtain the asymptotic solution. Moreover, the chaotic dynamics of the beam are discussed.

THEORETICAL BACKGROUND

An Euler-Bernoulli beam resting on the elastic foundation, with length L, width b and height h is considered. The beam is assumed to subject to a uniformly distributed harmonic excitation, $p(x,t) = P\cos\omega t$, with P and ω being the amplitude and frequency of the excitation, respectively. Based on the Newton's law of motion, considering the second-order moments of the subgrade reaction and excitation, we can obtain the nondimensional form of the motion equation

$$\ddot{v} + c\dot{v} + v'''' + K_0 v - K_1 v'' - J\ddot{v}'' = p\cos\Omega t \quad \begin{array}{l} -\frac{\lambda}{2}\left(v' + \frac{v'^3}{2}\right)' p\cos\Omega t - \frac{\lambda}{2}\left[v'(K_0 v - K_1 v'')\right]' \\ -\left[v'(vv'')'\right]' - \left(\frac{v'}{2}\int_1^x \frac{\partial^2}{\partial t^2}\int_0^x v'^2 \mathrm{d}x\mathrm{d}x\right)', \end{array} \tag{1}$$

where c is the damping coefficient; the prime and dot indicates differentiation with respect to x and t, respectively; and the nondimensional variables are introduced

$$x^*|v^* = \frac{x|v}{l}; \lambda = \frac{h}{l}; J = \frac{j}{ml^2}; p^* = \frac{Pl^3}{EI}; K_0 = \frac{k_0 l^4}{EI}; K_1 = \frac{k_1 l^2}{EI}; t^* = t\sqrt{\frac{EI}{ml^4}}; c^* = \frac{cl^2}{\sqrt{mEI}}; \Omega = \omega\sqrt{\frac{ml^4}{EI}}, \tag{2}$$

where $v(x,t)$ is the displacement of the beam along the x direction; $q(x,t) = k_0 v(x,t) - k_1 v''(x,t)$, where k_0 is the Winkler parameter, k_1 is the shear parameter; j is the rotary inertia; m is the the mass of the beam per unit length; E is the Young's modulus; I is the moment of inertia of the cross-section.

Similar to Lacarbonara et al.[2], we apply the state-space (first-order) form to investigate the nonlinear interaction of the beam on the elastic foundation. The full-basis discretization of the form is performed by letting $v(x,t) = \sum_{k=1}^{\infty} q_k(t)\phi_k(x)$ and $\dot{v}(x,t) = \sum_{k=1}^{\infty} z_k(t)\phi_k(x)$ ($k = 1, 2, \cdots, \infty$), where $q_k(t)$ and $z_k(t)$ are the unknown displacement and velocity coordinates, respectively; $\phi_k(x)$ is the kth in-plane mode shape corresponding to the natural frequency ω_k. Applying the Galerkin method, we can obtain

$$\begin{array}{ll} \dot{q}_k & -z_k = 0, \\ \dot{z}_k & +2\mu_k z_k + \omega_k^2 q_k = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{h=1}^{\infty}\Gamma_{kijh} q_i q_j q_h + \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{h=1}^{\infty}\Xi_{kijh} q_i(\dot{z}_j q_h + 2z_j z_h + q_j \dot{z}_h) \\ & + \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\Lambda_{kij} q_i q_j + \sum_{i=1}^{\infty}\Upsilon_{ki} q_i p\cos\Omega t + \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{h=1}^{\infty}\Theta_{kijh} q_i q_j q_h \cos\Omega t + f_k\cos\Omega t, \end{array} \tag{3}$$

where $\Upsilon_{ki} = -\frac{\lambda}{2}\int_0^1 \phi_i''\phi_k \mathrm{d}x$; $\Lambda_{kij} = -\frac{\lambda}{2}\int_0^1 (\phi_i'(K_0\phi_j - K_1\phi_j''))'\phi_k \mathrm{d}x$; $\Gamma_{kijh} = -\int_0^1 (\phi_i'\phi_j''\phi_h'' + \phi_i'\phi_j'\phi_h''')'\phi_k \mathrm{d}x$; $f_k = \int_0^1 p\phi_k \mathrm{d}x$; $\Xi_{kijh} = -\frac{1}{2}\int_0^1 (\phi_i'\int_1^x\int_0^x \phi_j'\phi_h' \mathrm{d}x\mathrm{d}x)'\phi_k \mathrm{d}x$; $\Theta_{kijh} = -\frac{\lambda}{4}\int_0^1 p(\phi_i'\phi_j'\phi_h')'\phi_k \mathrm{d}x$; $\mu_k = \frac{1}{2}\int_0^1 c\phi_k^2 \mathrm{d}x$.

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To describe the nearness to the internal resonance and primary resonance, the detuning parameters σ_1 and σ_2 are introduced such that $\omega_2 = 2\omega_1 + \varepsilon\sigma_1$ and $\Omega = \omega_2 + \varepsilon\sigma_2$. Following Lacarbonara et al. [2] and the method of multiple scales [3], we can obtain the following modulation equations

$$2\mathrm{i}\omega_1(\dot{A}_1 + \mu_1 A_1) = S_1 A_2 \bar{A}_1 \mathrm{e}^{\mathrm{i}\sigma_1 t} + S_{11} A_1^2 \bar{A}_1 + S_{12} A_1 A_2 \bar{A}_2, \tag{4}$$

$$2\mathrm{i}\omega_2(\dot{A}_2 + \mu_2 A_2) = S_2 A_1^2 \mathrm{e}^{-\mathrm{i}\sigma_1 t} + S_{21} A_2 A_1 \bar{A}_1 + S_{22} A_2^2 \bar{A}_2 + 0.5 f_2 \mathrm{e}^{\mathrm{i}\sigma_2 t}, \tag{5}$$

where A_1 and A_2 are the complex-valued amplitudes of the first and second modes; S_i are the *first-order nonlinear coefficients*. Introducing the polar transformations, $A_j = \frac{1}{2} a_j e^{\mathrm{i}\beta_j}$, $(j = 1, 2.)$, we can obtain the polar form of the modulation equations. Moreover, the second-order expansion of the displacements can be written

$$\begin{aligned} v(x,t) = {} & \varepsilon a_1 \cos(0.5\Omega t - 0.5\gamma_1 - 0.5\gamma_2)\phi_1 + \varepsilon a_2 \cos(\Omega t - \gamma_2)\phi_2 + \tfrac{\varepsilon^2}{2}\{a_1^2[\cos(\Omega t - \gamma_1 - \gamma_2)\psi_{11}(x) + \kappa_{11}(x)] \\ & + a_2^2[\cos 2(\Omega t - \gamma_2)\psi_{22}(x) + \kappa_{22}(x)] + a_1 a_2[\cos(1.5\Omega t - 1.5\gamma_1 - 0.5\gamma_2)\psi_{12}(x) + \cos(0.5\Omega t + 0.5\gamma_1 - 0.5\gamma_2)\kappa_{12}(x)]\}, \end{aligned} \tag{6}$$

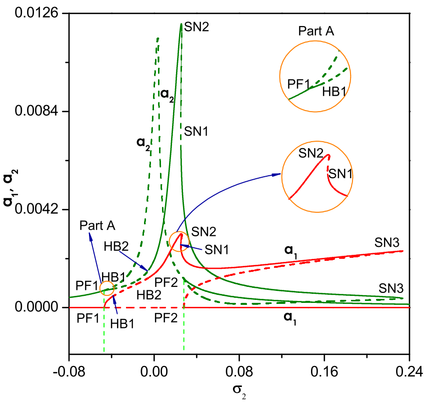

Figure 1. Frequency-response curves of the inextensional beam with $f_2 = 0.002$ when $\Omega \approx \omega_2$.

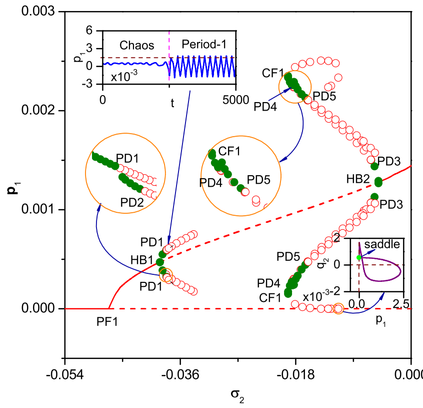

Figure 2. Periodic solution branch of the modulation equations when $\sigma_2 \in (-0.0391, -0.0042)$.

NUMERICAL RESULT AND DISCUSSION

In this study, we choose the following parameters and material properties of the beam: length $l = 6.096\mathrm{m}$, height $h = 0.305\mathrm{m}$, width $b = 0.61\mathrm{m}$, density $\rho = 2.403 \times 10^3 \mathrm{kg/m^3}$ and Young's modulus $E = 2.482 \times 10^4 \mathrm{Mpa}$. For the case of primary resonance of the second mode, Fig.1 shows the frequency-response curves of the inextensional beam, where HB, SN and PF denote the Hopf bifurcation point, saddle-node bifurcation point and pitchfork bifurcation point, respectively. The single-mode solution doesn't exhibit any multi-value region, and it loses stability via a pitchfork bifurcation at PF1, then regains stability via another pitchfork bifurcation at PF2. Whereas, the indirectly excited mode may dominate the nonlinear response in the second region. Moreover, the two-mode solution branch exhibits the existence of two Hopf bifurcations at HB1 and HB2. Fig.2 shows the periodic solution branches emerged from HB1 and HB2, where PD and CF denote the period-doubling bifurcation point and cyclic-fold bifurcation point, respectively. As shown, the periodic solutions, starting from HB1 and HB2, lose stability via periodic-doubling bifurcations at PD1 and PD3. When decreasing σ_2, the latter regains stability at PD4, and a cyclic-fold bifurcation occurs at CF1. Continuing the periodic solution from CF1, we note that the lower branch approaches the saddle, and the unstable homoclinic orbit can be obtained.

CONCLUSIONS

The multimode nonlinear response of the beam resting on the elastic foundation due to the two-to-one internal resonance has been investigated in this study. The beam is subjected to the primary resonance of the second modes. The method of multiple scales has been applied to determine the modulation equations governing the two-to-one internal resonant interaction and the displacement of the beam. Numerical results have been obtained to explore the nonlinear dynamics of the beam.

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