OPTIMAL CONTROL OF VIBRATIONALLY EXCITED LOCOMOTION SYSTEMS

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Summary Optimal control problems are considered for locomotion systems that can move in resistive media without special propelling devices, such as legs, wheels or caterpillars, due to the periodic change in their configurations. This principle of motion is based on relative vibrations of the system's components. Design and control variables are selected to maximize the average speed of the system in a desired direction .

OPTIMAL MOTION OF MULTI-LINK LOCOMOTION SYSTEMS IN A FLUID

The planar motion of a mechanical system that consists of the main body and two identical links OA and $O'A'$ attached to it by revolute joints O and O' (Fig. 1) is considered.

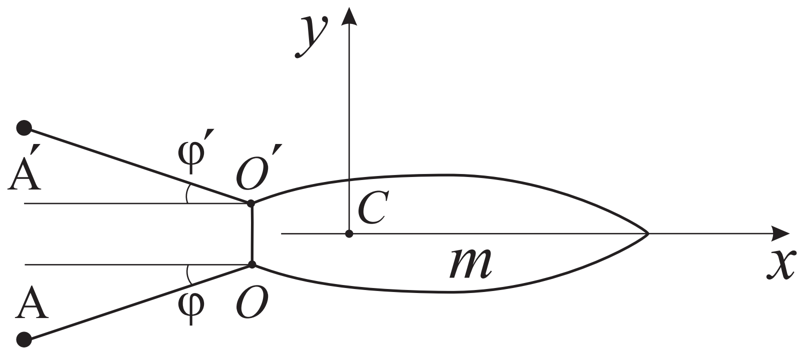

Figure 1. System with two links

The shape of the main body is assumed to be symmetric with respect to the plane that passes through the longitudinal axis of the body and is perpendicular to the plane of motion. The axes of the joints are arranged on the main body symmetrically relative to its symmetry plane. The links perform periodic angular oscillations with respect to the main body in such a way that the configuration of the system always remains symmetric with respect to the x-axis. If at an initial time instant the main body moves translationally along the x-axis, then it will move translationally along this axis later. This mode of motion imitates swimming of some aquatic animals, e.g., frogs.

The motion of the main body generated by periodic vibrations of the links is studied. It is assumed that (i) the force applied to a moving body is directed against the velocity of the body relative to the medium and the magnitude of this force is a quadratic function of the absolute value of this velocity (quadratic friction law), (ii) the resistance forces acting on the links are applied at the points A and A', and (iii) during the period T, the angle φ first increases from 0 to φ_0 and then decreases from φ_0 to 0.

A mathematical model for the system under consideration is derived. It is shown that if the angle φ increases and decreases at constant rate, then for the main body to move progressively with periodically changing velocity and positive average speed, it is necessary and sufficient that the rate of motion of the links away from the main body axis be less than the rate of motion of the links toward this axis.

The optimal laws for the motion of the links that maximize the average speed of the main body are found. To this end, an optimal control problem is solved. The rate of rotation of the links relative to the main body is used as the control variable, which is allowed to change within a prescribed interval. The maximum angle φ_0 of deviation of the links from the axis of the main body is fixed. The optimal mode of oscillations of the links is close to their motion with piecewise constant angular velocity. The piecewise constant law for the angular velocity of the links can be regarded as a near-optimal control. The difference between the optimal and near-optimal control laws in terms of the average speed of the main body is less than 2%.

Similar optimal control problems are solved also for the motion in media characterized by more general nonlinear resistance laws than the quadratic friction.

Apart from the system with two links shown in Fig. 1, a system with one link attached to the main body is studied. In this system, the axis of rotation of the link is perpendicular to the x-axis, and to compensate the rotation of the main body for the period, the link rotates alternately to both sides of the x-axis. This system imitates swimming of fishes.

The simple models described above take into account only the drag force that is directed against the velocity of motion of

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a body through the medium. More complicated models that take into account the lift forces applied to the links are also considered. It is shown that the presence of the lift forces always leads to an increase in the body speed.

OPTIMAL CONTROL OF A LOCOMOTION SYSTEM WITH A MOVABLE INTERNAL BODY

A two-body locomotion system shown in Fig. 2 is considered.

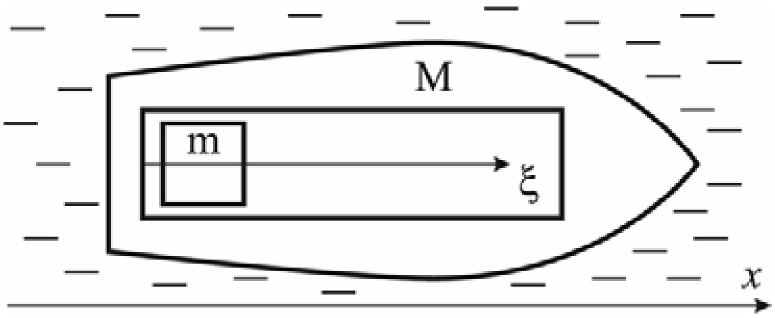

Figure 2. Two-body locomotion system

The system consists of the main body M that interacts with the medium and an auxiliary body m that can move relative to the main body, interacts with it but does not interact with the medium. This auxiliary body will be referred to as the internal body, although it is not necessary that this is located inside the main body. The internal body interacts with the main body by means of the force produced by the drive. The driving force applied to the internal body causes the reaction force applied to the main body. The reaction force changes the velocity of the main body relative to the medium, which leads to a change in the force of resistance of the external medium. Therefore, by controlling the motion of the internal body, one can change the external force acting on the main body, thus controlling the motion of the entire system. This principle of motion is utilized for certain types of mini-sized locomotion robots. The main body of such robots can be made hermetic and smooth, without any protruding parts, which enables these robots to be used for nondestructive inspection of mini-sized engineering objects (e.g., thin-walled pipelines of small diameter), as well as in medicine.

We consider a simple model of the system with the internal body moving relative to the main body translationally along a straight line. The main body is assumed to move relative to the medium along the same line. For this model, it is required to find a periodic motion of the internal body relative to the main body that generates the motion of the main body relative to the medium with periodically changing velocity and provides a maximum for the displacement of the main body for the period. The period of the relative motion of the internal body is assumed to be prescribed and equal to the period of change of the velocity of the main body. The acceleration of the internal body relative to the main body or the force of interaction of these bodies is used as the control variable. Qualitative characteristics of the optimal motion, such as the number of switchings of the control function during the period, the absence or presence of a singular control mode, in which the control variable takes on an intermediate value between the minimum and maximum allowed values, the conditions for entering and leaving the singular mode, are investigated. It is proved that the singular mode necessarily occurs in the optimal motion, if the resistance law is an odd function of velocity and is either convex or concave for the velocity constant in sign. Such resistance laws are widely used for modeling the resistance of viscous media to the motion of rigid bodies. For this class of resistance laws (that include quadratic friction), the algorithms for calculating the optimal control are reduced to solving systems of finite equations for the control parameters. Illustrative examples are given.

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