TWO PARAMETER CODIMENSION-THREE BIFURCATIONS OF A NOVEL NONLINEAR OSCILLATOR WITH STRONG IRRATIONAL NONLINEARITIES

Qingjie Cao[*a)], Yanwei Han[*], Yushu Chen[*] Marian Wiercigroch[**] & Sergey Piskarev[***]

[*]*School of Astronautics, Harbin Institute of Technology, Harbin, 150001, China*

[**]*Centre for Applied Dynamics Research, Department of Engineering, University of Aberdeen, King's College, Aberdeen AB24 3UE, Scotland, UK*

[***]*Department of Mathematics, Moscow State University, Moscow, Russia*

Summary   In this paper, we investigate the two parameter codimension three degenerate bifurcations of a novel model based upon the recently proposed smooth and discontinuous (SD) oscillator [1]-[4] having strong irrational nonlinearities due to the geometrical configuration. The investigation shows that the system admits the complicated two parameter codimension three bifurcations near the catastrophe point C in parameter (α, β) space. It is found that the universal unfolding of the system is equivalent to the system perturbed by a nonlinear viscous damping. The bifurcation diagrams and the corresponding codimension three behaviours are obtained and presented by employing subharmonic Melnikov functions for the existed singular closed orbits of homoclinic, tangent homoclinic, homo-heteroclinic and cuspidal heteroclinic, respectively.

INTRODUCTION

Consider the two parameter codimension three degenerate bifurcations of a nonlinear vibration, proposed in [5], written as the following form.

$$\ddot{x} + F(x, \alpha, \beta) = 0, \tag{1}$$

where $\dot{x} = \mathrm{d}x/\mathrm{d}\tau$, $F(x, \alpha, \beta) = (x+\alpha)\,(1 - 1/\sqrt{(x+\alpha)^2+\beta^2}) + (x-\alpha)\,(1-1/\sqrt{(x-\alpha)^2+\beta^2})$. This system is a strong irrational nonlinear possessing of both smooth ($\beta > 0$) and discontinuous ($\beta = 0$) behaviours depending on the value of parameter, β. There is little possibility to get the analytical expressions, the details seen in Appendix A of [2], for both equilibria and the separatrices at the smooth regime and the lose of efficacy of the conventional procedures due to barrier of the Taylor expansion at the discontinuous phase. In this paper, both equilibrium stability and separatrix bifurcations are analysed topologically at the smooth regime near the catastrophe point $C\left(4\sqrt{5}/25, 8\sqrt{5}/25\right)$ by introducing the Taylor expansion for the resistance force, written as.

$$F(x,\alpha,\beta) = 2x - \sum_{n=0}^{\infty}[\frac{P_n(\alpha)}{\beta^{n+1}}(x+\alpha)^{n+1} + \frac{P_n(-\alpha)}{\beta^{n+1}}(x-\alpha)^{n+1}],$$

where $P_n(x) = \dfrac{1}{2^n n!}[\dfrac{\mathrm{d}^n}{\mathrm{d}x^n}(x^2-1)^n], n = 0, 1, 2, 3, \cdots$. Truncating the above series to the fifth order, leads to the equivalent system of equation (1) as the following form.

$$\ddot{x} + C_1(\alpha,\beta)x + C_3(\alpha,\beta)x^3 + C_5(\alpha,\beta)x^5 = 0, \tag{2}$$

Setting $\tau = \sqrt{C_5}t$, follows that system (2) can be re-written as the form.

$$\ddot{x} + \varepsilon_1 x + \varepsilon_3 x^3 + x^5 = 0. \tag{3}$$

which is a two parameter system behaving codimension three bifurcation phenomena, where $\varepsilon_1 = C_1/C_5, \varepsilon_1 = C_3/C_5$. It is worth noticing that the following cases are of interesting for system (3) having cuspidal heteroclinic, tangent homclinic, homclinic and homo-heteroclinic orbits, respectively.

1. $\varepsilon_2 = -2\sqrt{\varepsilon_1}$, for the cuspidal heteroclinic orbit;
2. $\varepsilon_1 = 0, \varepsilon_2 < 0$, for tangent homclinic orbit;
3. $\varepsilon_1 < 0$, for homoclinic orbit, and
4. $\varepsilon_2 < -2\sqrt{\varepsilon_1}$ for homo-heteroclinic orbit.

CODIMENSION-THREE BIFURCATIONS

It is assumed that the system (3) is perturbed by a general Van der Pol nonlinear damping $(\xi + \eta x^2 + \zeta x^4)y$ [6, 7]. This leads to the forced dissipative oscillator of the following form:

$$\ddot{x} + (\xi + \eta x^2 + \zeta x^4)y + \varepsilon_1 x + \varepsilon_3 x^3 + x^5 = 0. \tag{4}$$

a) Corresponding author. E-mail: q.j.cao@hit.edu.cn.

Re-scaling system (4) and letting $\dot{x} = y$, lead to the following system.

$$\begin{cases} \dot{x} = y, \\ \dot{y} = \mu_1 x + \mu_2 x^3 - x^5 + \mu_3 y + \mu_4 x^2 y - x^4 y, \end{cases} \tag{5}$$

to which the Melnikov function can be employed to get the bifurcation phenomena for all the cases listed above. Without loss of generality, we only present the results here for the double well dynamics as an example at the end of this paper. The transition set, seen the left part of the Figure 1, can be obtained and written as the following for the specialised case of homo-heteroclinic orbits.

$$\mathrm{H} = \mathrm{H}_2 \cup \mathrm{HL} \cup \mathrm{B} = \{\mu_3 = 2.0\mu_1\} \cup \{\mu_3 = 1.4\mu_1\} \cup \{\mu_3 = 1.31\mu_1\}, \tag{6}$$

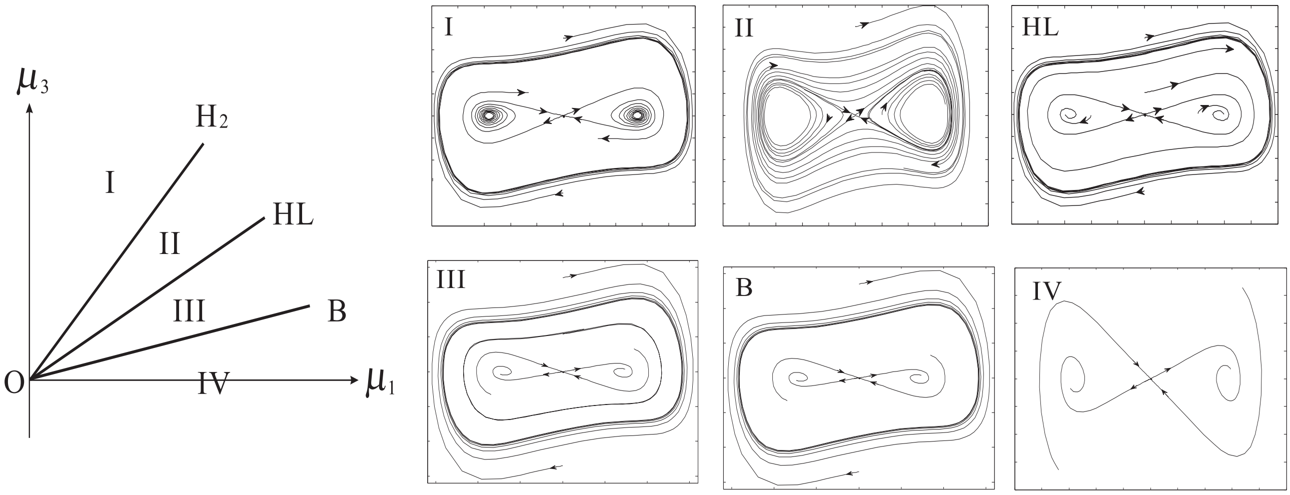

Figure 1. The bifurcation sets and the associated phase portraits of system (5)

which divides parameter plane (μ_1, μ_3) into four persistent regions, I, II, III and IV, in each of these regions system (5) is structurally stable. The corresponding phase portraits, both the persistent and the nonpersistent, are plotted, respectively, on the right part of Figure 1, which show the Hopf bifurcations, closed orbit bifurcation and also the homo-heteroclinic bifurcations.

CONCLUSIONS

In this paper, we have investigated the codimension three bifurcation phenomena of a strong irrational nonlinear oscillator with two bifurcation parameters. It has been found that the universal unfolding of the system near catastrophe point C, at which codimension three phenomena occur, is the perturbed system with nonlinear damping. Subharmonic Melnikov method has been employed to obtain the complex bifurcations, i.e., Hopf bifurcations, homo-heteroclinic bifurcation and the closed orbit bifurcation.

ACKNOWLEDGMENTS

The first two author acknowledges the financial support from National Science Foundation of China, grant 10872136 and 11072065.

References

[1] Cao Q. J., Wiercigroch M., Pavlovskaia E. E., Grebogi C., Thompson J. M. T.: Archetypal Oscillator for Smooth and Discontinuous Dynamics. *Phys. Rev. E*, **74**: 046218, 2006.

[2] Cao Q. J., Wiercigroch M., Pavlovskaia E. E., Grebogi C., Thompson J. M. T.: Piecewise Linear Approach to an Archetypal Oscillator for Smooth and Discontinuous Dynamics. *Phil. Trans. R. Soc. A*, **366**: 635-652, 2008.

[3] Cao Q. J., Wiercigroch M., Pavlovskaia E. E., Grebogi C., Thompson J. M. T.: The Limit Case Response of the Archetypal Oscillator for Smooth and Discontinuous Dynamics. *Int. J. Nonlin. Mech.*, **43**: 462-473, 2008.

[4] Cao Q. J., Xiong Y. P., Wiercigroch M.: Resonances Behavior of SD Oscillator at the Discontinuous Phases. *J. Appl. Anal. Compu.*, **1**: 183-191, 2011.

[5] Han Y. W., Cao Q. J., Chen Y. S., Wiercigroch M.: A Novel Smooth and Discontinuous Oscillator With Strong Irrational Nonlinearities. *Sci. China Ser. G*, 2011 (accepted).

[6] Tian R. L., Cao Q. J., Li Z. X.: Hopf Bifurcations for the Recently Proposed Smooth-and-Discontinuous Oscillator. *Chin. Phys. Lett.*, **27**: 074701, 2010.

[7] Tian R. L., Cao Q. J., Yang S. P.: The Codimension-two Bifurcation for the Recent Proposed SD Oscillator. *Nonlin. Dyn.*, **59**: 19-27, 2010.