Optimal bounded control of the randomly excited nonlinear oscillators containing fractional derivative damping

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Summary The stochastic dynamical systems with fractional derivative damping have been received much attention in the past decades, e.g., see Refs.[1,2]. However, the optimal control of randomly excited systems containing fractional derivative damping has never been studied so far. On the other hand, feedback optimal control of stochastic dynamical systems has been an attractive field, but still remains an open challenge. Zhu and his co-workers [3] have proposed a nonlinear stochastic optimal control strategy for the control of mechanical and structural systems under random excitations, and this strategy exhibits good control performance either in effectiveness or in efficiency than LQG control strategy. In this paper, a procedure for designing optimal bounded control to minimize the response of randomly excited nonlinear oscillators containing fractional derivative damping is addressed. Two examples are given to illustrate the application of the proposed procedure and the effectiveness of control on the response of the system.

1. Problem formulation

Consider a controlled single-degree-of-freedom nonlinear oscillator with fractional derivative damping subjected to Gaussian white noise excitations. The equation of motion is

$$\ddot{X}+C(X,\dot{X})D^{\alpha}X(t)+g(X)=u(X,\dot{X})+f_k(X,\dot{X})\xi_k(t) \tag{1}$$

where $CD^{\alpha}X(t)$ is the small damping term, in which $D^{\alpha}X(t)$ denotes the fractional derivative operation of Caputo's definition, i.e.,

$$D^{\alpha}X(t)=\frac{1}{\Gamma(1-\alpha)}\int_0^t\frac{\dot{X}(t-\tau)}{\tau^{\alpha}}\mathrm{d}\tau\ ,\ 0<\alpha<1 \tag{2}$$

$\Gamma()$ is gamma function and α is the fractional order; $g(X)$ is the linear and/or nonlinear restoring force; u denotes the weak feedback control; f_k denote the amplitudes of weakly excitations; $\xi_k(t)$ are Gaussian white noises with correlation functions $E[\xi_i(t)\xi_j(t+\tau)]=2D_{ij}\delta(\tau)$, $i,j=1,2,\ldots,l$. Followed the stochastic averaging method proposed in Ref.[2], the Eq.(1) is converted into the partially averaged Itô equation for amplitude,

$$\mathrm{d}A=[\bar{m}_1(A)+\bar{m}_2(A,u)]\mathrm{d}t+\bar{\sigma}_1(A)\mathrm{d}B(t) \tag{3}$$

Note that the detailed derivations and expressions for $\bar{m}_1$ and $\bar{b}_1$ can be found in Ref.[2] and the term containing control force will be averaged later.

2. Nonlinear stochastic optimal control

In this section, we are interested in the ergodic control of the response in a finite or semi-infinite time interval. Thus, the following form of a performance index is considered:

$$J=\lim_{T\to\infty}\frac{1}{T}\int_0^T<f(A(s),u(s))>\mathrm{d}s \tag{4}$$

By applying the stochastic dynamical programming principle to Eq.(3), the dynamical programming equation is derived

$$\lambda=\min_u\{f(A,u)+[\bar{m}_1(A)+\bar{m}_2(A,u)]\frac{\partial V}{\partial A}+\frac{1}{2}\bar{b}_{11}(A)\frac{\partial^2V}{\partial A^2}\} \tag{5}$$

where $V=V(A)$ is the value function and λ is the optimal performance index . The optimal control law can be determined from minimizing the right hand side of Eq.(5) with respect to u under control constraint. Assume that the control constraint is of the form $|u|\le u_0$, where u_0 is a positive constant and represents the maximum control force. In this case, the performance index (4) does not depend explicitly on u, i.e., $f(A,u)=f(A)$. Obviously, when $u_{\mathrm{opt}}=-u_0\,\mathrm{sgn}(\dot{X}\,\partial V/\partial A)$,the right hand side of Eq.(5) will be minimum. In addition, take $f(A)$ so that $\partial V/\partial A$ is usually positive. Therefore, the optimal control is reduced as $u_{\mathrm{opt}}=-u_0\mathrm{sgn}(\mathrm{d}X/\mathrm{d}t)$, which implies that the optimal control is a bang-bang control. Inserting u_{opt} into partially averaged equation for replacing u and complete averaging, one can obtain the fully averaged Itô equation.The approximate probability density and statistics of stationary response of controlled system (1) can be then obtained by solving the stationary FPK equation associated with the fully averaged Itô equation

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3. Examples

In this section, two examples are worked out. The first one is the controlled Duffing oscillator. The motion equation is of the form

$$\ddot{X}(t)+\beta D^{\alpha}X(t)+\omega_0^2 X+\alpha_0 X^3 = u+\xi(t) \tag{6}$$

Some numerical results for $p(A)$,$p(X_1)$ and $E[A]$of parameters β=0.05,α=0.15,ω_0=1,α_0=2 and D=0.01, are shown in Figs.1-3. It is seen that the analytical shown by solid lines and the simulation results denoted by symbols (●,▲,■) are good in agreement and the control force really alleviates the stationary response of system.

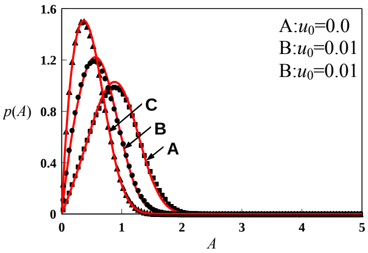

Fig.1

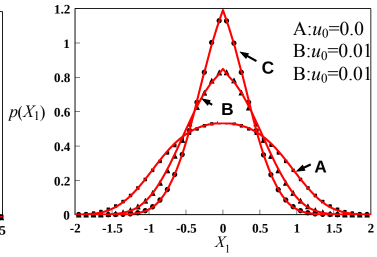

Fig.2

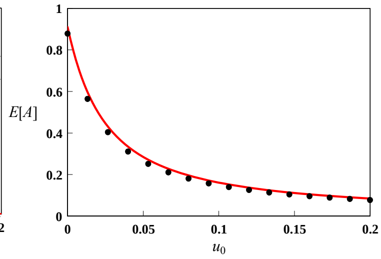

Fig.3

As the second example, a controlled ver de Pol oscillator is considered. The equation of the motion is of the form

$$\ddot{X}+(-\beta_0+\beta_1 X_0^2)D^{\alpha}X+\omega_0^2 X = u+\xi(t) \tag{7}$$

Some numerical results for $p(A)$,$p(X_1)$ and $E[A]$of parameters β_0=β=0.05, α=0.15, ω_0=1 and D=0.01are shown by solid lines in Figs.4-6.The corresponding results obtained by using simulation are also shown by symbols (●,▲,■) in these figures. It is seen that the analytical results and those from simulation are in very good agreement and the stationary response is alleviated greatly by the control. Besides, Fig.5 shows that control force play significant roles in the system response behavior, a diffusive limit cycle is found in uncontrolled system and then disappears when control force enlarges.

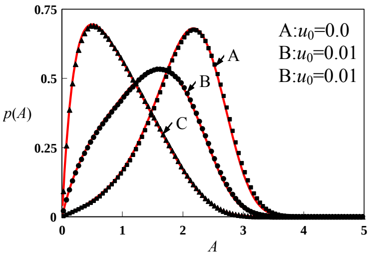

Fig.4

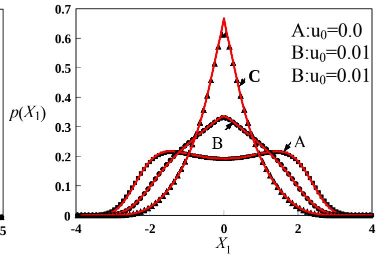

Fig.5

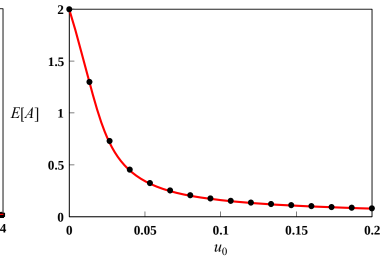

Fig.6

4. Conclusions

In this paper, a procedure for designing the optimally bounded control of randomly excited nonlinear oscillators with fractional derivative damping for minimizing the response has been developed. The procedure consists of applying the stochastic averaging method, establishing the dynamical programming equations from the partially averaged equations, determining the optimal control from the dynamical programming equations and control constraints, and obtaining the response of controlled systems from solving the FPK equation associated with the fully averaged systems. By applying the proposed procedure to two simple examples, it is shown that the stationary response may be alleviated greatly by the control.

Acknowledgements

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