ON ROBUST DESIGN OPTIMIZATION OF TRUSS STRUCTURE WITH BOUNDED UNCERTAINTIES

Zhan Kang[a)], Song Bai
State Key Laboratory of Structural Analysis for Industrial Equipment,
Dalian University of Technology, Dalian, 116024, China

Summary This paper investigates robust design optimization of truss structures with uncertain-but-bounded parameters and loads. The variation of the Young's moduli is treated with non-probabilistic ellipsoid convex models. A robustness index for quantitatively measuring the maximal allowable magnitude of system variations is presented, and the design problem is then formulated as to maximize the minimum of the robustness indices for all the concerned design requirements under a given material volume constraint.

INTRODUCTION

Previous study has shown that uncertainties may cause significant variability in structural performance, thus they need to be properly accounted for in the optimal structural design problem [1]. In practical engineering problems, the bounds of the uncertain parameters can be more easily obtained. The convex models are well suited for describing such kind of unknown-but-bounded parameters. Ellipsoidal convex models have been used in many non-deterministic structural optimization problems in the context of robust design [2, 3].

In this work, following the concept that a structural design is deemed more robust if it can sustain a wider range of uncertain variations, we study the formulation and numerical techniques for robust design optimization of truss structures. The ellipsoid convex model is employed for describing uncertain-but-bounded loads and parameters. A corresponding robustness index is proposed and it is treated as the objective function in the optimal design problem.

STRUCTURAL ROBUSTNESS INDEX

The convex set $\mathbb{E}(\xi)$ can be expressed in form as

$$\mathbb{E}(\xi)=\left\{\mathbf{y}\in\mathbb{R}^n \mid \mathbf{y}=\left(\mathbf{I}+\mathrm{diag}\left(\mathbf{z}\right)\right)\bar{\mathbf{y}},\ \mathbf{z}=\mathbf{T}\boldsymbol{\varphi},\ \left\|\boldsymbol{\varphi}\right\|_2\le\xi,\ \boldsymbol{\varphi}\in\mathbb{R}^n\right\}$$

where the scalar ξ and the matrix $\mathbf{T}$ respectively determine the size, the aspect ratio and the orientation of the ellipsoid. In a structural design problem, we define the status of a structural performance by a feasibility function $g\left(\mathbf{x},\boldsymbol{\varphi}\right)$, in which $\mathbf{x}$ represents the design variable vector and $\boldsymbol{\varphi}$ the uncertainty vector.

The normalized uncertainty space is divided into two domains by the feasibility function, namely a feasible domain $\left\{\boldsymbol{\varphi}\in\mathbb{R}^n \middle| g\left(\mathbf{x},\boldsymbol{\varphi}\right)>0\right\}$ and a non-feasible domain $\left\{\boldsymbol{\varphi}\in\mathbb{R}^n \middle| g\left(\mathbf{x},\boldsymbol{\varphi}\right)<0\right\}$, and the critical surface (or critical curve in 2-D cases) is defined by $g\left(\mathbf{x},\boldsymbol{\varphi}\right)=0$.

For a certain design $\mathbf{x}$, one can find a circle that is tangential to the critical curve $g\left(\mathbf{x},\boldsymbol{\varphi}\right)=0$, and the radius of this circle as well as the tangential point can be found by seeking the closest point between the critical curve and the origin of the normalized uncertainty space. If the circle is further expanded, non-feasible combinations of uncertain parameters will be included. Thus, the circle that is tangential to the critical curve represents the maximum allowable variation of the structural system for a fixed design, and the radius ξ of this circle is defined as the robustness index.

NUMERICAL EXAMPLE

The robust optimization of a ten-bar truss structure exhibiting material property uncertainty is considered in this example. The structure and the loading condition are shown in Fig. 1. Two concentrated forces $\overline{P_1}=100$ and $\overline{P_2}=100$ are applied at node 6 and node 4, respectively. We divide the bars into two groups (see Table 1) and suppose that all the bars within each group have the same Young's modulus. The Young's moduli of both groups of bars $\overline{E_1}=10000$ and $\overline{E_2}=8000$ are unknown-but-bounded parameters. The uncertain Young's moduli are modeled by an ellipsoid model represented by $\mathbf{z}\in\left\{\mathbf{z}\in\mathbb{R}^2 \mid \mathbf{z}=\xi\mathbf{T}\boldsymbol{\varphi},\ \left\|\boldsymbol{\varphi}\right\|_2\le 1,\ \boldsymbol{\varphi}\in\mathbb{R}^2\right\}$, in which the transformation matrix is

$$\mathbf{T}=\begin{pmatrix}-0.09998 & 0.00275\\ 0.00229 & 0.08329\end{pmatrix}$$

[a)] Corresponding author. Email: zhankang@dlut.edu.cn.

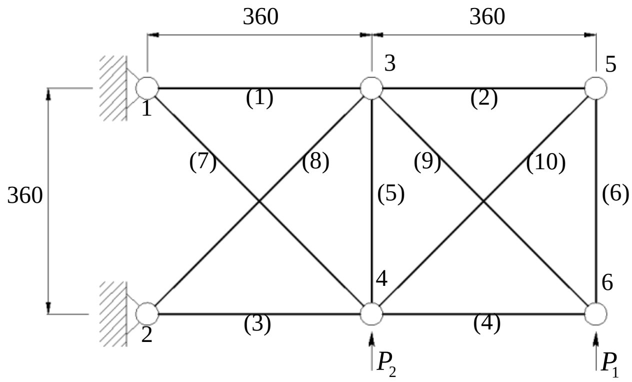

Fig. 1. A ten-bar planar truss subjected to concentrated forces

The vertical displacements at node 3 and node 6 are restricted by $v_3 \le 2.0$ and $v_6 \le 5.0$ respectively, the structural volume is constrained as $V \le 22700$. The cross-sectional areas of the member bars are considered as design variables, with lower limits $\underline{a}_i = 0.01$, upper limits $\bar{a}_i = 40.00$ and initial values $a_i = 20.00$ $(i = 1, 2, ..., 10)$. The optimal results are listed in Table 1.

Table 1 Optimal solutions for the ten-bar truss

Member number	Cross-sectional area		
	Initial design	Optimal design	
		Deterministic	Robust design
1	7	13.06	14.03
2	7	0.01	0.01
3	7	9.04	9.71
4	7	6.48	6.96
5	7	0.01	0.01
6	7	0.01	0.01
7	7	0.71	0.73
8	7	10.44	11.16
9	7	10.24	10.95
10	7	0.01	0.01
$\xi_{v_3}^*$	-5.53	1.06	1.90
$\xi_{v_6}^*$	-3.46	0.00	1.00
Volume	29375	21195	22700

CONCLUSIONS

This paper investigates robust optimization of truss structures with unknown-but-bounded parameters using ellipsoid convex models. Therein, a quantified measure of robustness is proposed, and the design objective is to maximize the minimum one of the robustness indices for the concerned structural performance requirements. Numerical example of truss structures were treated with the present formulation, and the results showed the validity of the proposed method.

References

[1] Schueller G.I., Jensen H.A.: Computational methods in optimization considering uncertainties - An overview. *J. Comput Meth Appl Mech Eng.* **198**: 2-13, 2008.

[2] Ben-Tal A., Nemirovski A.: Robust truss topology design via semidefinite programming. *J. SIAM J Optim*. **7**:991-1016, 1997.

[3] Yonekura K., Kanno Y.: Global optimization of robust truss topology via mixed integer semidefinite programming. *J. Optim Eng.* **11:**355-79, 2010.