Structural topology optimization based on analysis mesh-independent density interpolation

Zhan Kang [*], Yiqiang Wang
[*]*State Key Laboratory of Structural Analysis for Industrial Equipment, Dalian University of Technology, Dalian 116024, China*

Summary A non-local density interpolation strategy for topology optimization based on nodal design variables is proposed. Design variable points can be positioned at any locations, and thus separated from finite elemental nodes. The density value of any computational point can be computed by design variable points within the influence domain of the point by the use of the Shepard interpolants, which strictly satisfy range-restricted properties. A numerical example is presented to demonstrate the validity of the proposed method and numerical techniques.

INTRODUCTION

Topology optimization is considered as one of the most challenged technologies in the optimal field, and has seen a rapid development during the past two decades. Many methods have been proposed according to different mathematical description models, such as the SIMP method [1], and the level set-based method [2]. Recently, topology optimization methods based on nodal design variables have attracted much attention [3,4].
This work proposes a nodal design variable-based topology optimization method. The basic idea is to construct the density field from design variable points located within a certain influence domain of each computational point, rather than within each element, using the Shepard interpolants. By this means, the density field can be represented in a completely element-independent manner. Meanwhile, the proposed material density interpolation scheme strictly satisfies the [0, 1] range-restricted properties required by a physically significant density interpolation. As a consequence, higher-order finite elements can be applied in discretization of the design domain. The proposed method is shown to be capable of alleviating numerical instabilities such as checkerboard patterns, mesh dependence and “islanding” phenomenon without employing artificial filtering or other additional measures. With different cut-off radius values and mesh resolutions, the method can yield basically similar optimal topologies.

COMPLIANCE MINIMIZATION PROBLEM

In this study, the minimum compliance problem is considered by using nodal density design variables and non-local Shepard interpolants. The design variables points can be positioned freely in the design domain. The interpolation functions are given by

$$\omega_i(x) = \frac{D(x-x_i)}{\sum_{j\in S_x} D(x-x_j)} \qquad (i \in S_x)$$

where S_x is the influence domain of the point x, and $D(x-x_i)$ is the inverse of square of the distance from point x to point x_i. The density value at point x can be expressed as

$$\rho(x) = \sum\nolimits_{i\in S_x} \omega_i(x)\rho_i \tag{1}$$

where ρ_i is density value at the ith design variable point. By using the finite element discretization, the minimum compliance problem for the linear structure is stated as

$$\begin{aligned} &\underset{\rho_i\ (i=1,2,\ldots,N)}{\text{Minimize}} \quad c = \mathbf{f}^{\mathrm{T}}\mathbf{u} \\ &\text{Subject to} \quad \begin{cases} \mathbf{Ku} = \mathbf{f} \\ V - f_{\mathrm{v}}V_0 \le 0 \\ 0 < \underline{\rho} \le \rho(x) \le 1 \end{cases} \end{aligned} \tag{2}$$

where $\mathbf{f}$, $\mathbf{u}$ and $\mathbf{K}$ are the external force vector, the displacement vector and the stiffness matrix, respectively, V is the material usage and V_0 is the volume of design domain.
Applying the adjoint sensitivity analysis, the derivative of the elemental stiffness matrix with respect to ρ_i can be computed as

$$\frac{\partial \mathbf{K}_e}{\partial \rho_i} = \begin{cases} \int_{\Omega_e} p\rho^{p-1}(x)\omega_i(x)\mathbf{B}^{\mathrm{T}}\mathbf{D}_0\mathbf{B}d\Omega & i \in S_x \\ 0 & i \notin S_x \end{cases} \tag{3}$$

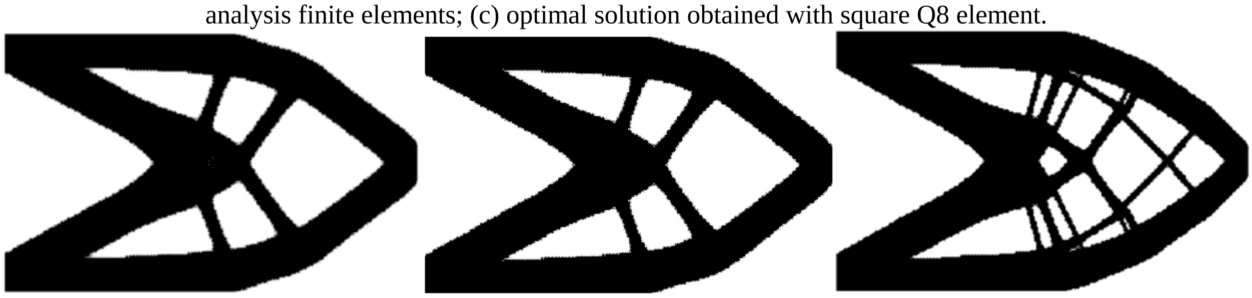
NUMERICAL EXAMPLE

A well-known cantilever beam design problem is considered. The geometrical dimension of the design domain is 40×25. A unit concentrated load is applied to the mid-point of the right free edge. The design domain is discreted into 17784 three-node unstructured (T3) finite elements. There are 121×76 design variable points distributed uniformly within design domain. It should be noted that the finite element nodes and the design variable points do not coincide with each other. The cut-off radius is set to be $R_c = 1$.

The optimal solution attained by the proposed method is presented in Figure 1(a). This solution exhibits no checkerboard patterns or "islanding" phenomenon. The optimal material distribution is not strictly symmetrical and this is due to use of the unstructured mesh. The procedure converges to the value of 0.3912 at the 60th step. Two other analysis meshes are also considered, which contain 240×150 right-angled T3 elements and 120×75 square Q8 elements, respectively. The final topologies of both meshes are presented in Figure 1(b) and Figure 1(c). It is found that the optimal results obtained with three different analysis meshes are very similar. Topological solutions with different values of the cut-off radius are also tested. Here, the design domain is discretized by 120×75 uniform-sized Q8 elements and the design variable points coincide with the finite element nodes. The optimal results obtained with cut-off radius values $R_c = 0.67$, 0.495 and 0.25 are given in Figure 2. These results also exhibit no checkerboard patterns and "islanding" phenomenon. When the radius is larger than the elemental size ($R_c/h \geq 1$), the optimized topologies are almost the same. On the other hand, more details are revealed by the optimal solutions obtained with a cut-off radius smaller than the element size ($R_c / h < 1$).

(a) (b) (c)

Figure 1. Topology optimization results on each refinement level. (a) optimal solution of material density field interpolated by design variables with unstructured T3 analysis finite elements; (b) optimal solution with structured T3 analysis finite elements; (c) optimal solution obtained with square Q8 element.



(a) (b) (c)

Figure 2. Optimization results obtained from different cut-off radius. (a) $R_c / h = 2$; (b) $R_c / h = 1.5$; (c) $R_c / h = 0.75$.

CONCLUSIONS

A topology optimization method based on nodal design variables with the non-local interpolation is presented. The Shepard interpolation scheme, which has a strictly [0, 1] range-restricted property, is used for constructing the material density field. The numerical example confirms that the proposed method alleviates numerical instabilities, such as the checkerboard patterns and "islanding" phenomenon. In particular, a smaller cut-off radius usually leads to a final structural layout with more fine details, while a larger cut-off radius tends to smear out thin members but still provides a similar overall structural layout from conceptual design point of view.

References

[1] Rozvany GIN, Zhou M, Birker T. Generalized shape optimization without homogenization. *Struct Multidiscip O* **4** : 250-254,1992.

[2] Wang MY, Wang XM, Guo DM.: A level set method for structural topology optimization. *Comput Method Appl Mech Engrg* **192**:227-246, 2003.

[3] Matsui K, Terada K.: Continuous approximation of material distribution for topology optimization. *Int J Numer Meth Engrg* **59**:1925-1944, 2004.

[4] Kang Z, Wang Y. Structural topology optimization based on non-local Shepard interpolation of density field. *Comput Method Appl Mech Engrg* **200**:3515-3525, 2011.