TOPOLOGY OPTIMIZATION OF DAMPING LAYER IN SHELL STRUCTURES

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Summary This paper investigates the optimal distribution of damping material in vibrating structures subject to harmonic excitations by using topology optimization method. Therein, the design objective is to minimize the structural vibration level at specified positions by distributing a given amount of damping material. The vibration equation of the structure has a non-proportional damping matrix. A system reduction procedure is first performed by using the eigenmodes of the undamped system. The complex mode superposition method in the state space is then employed to calculate the steady-state response of the vibrating structure. In this context, an adjoint variable scheme for the response sensitivity analysis is developed.

TOPOLOGY OPTIMIZATION PROBLEM FORMULATION

In this study, we consider the topology optimization problem for layout design of the damping material surface layer attached to a thin-shell base structure with the aim to reduce the vibration amplitude under harmonic excitations. Such a structure is schematically illustrated in Fig. 1. It is assumed that the damping layer and the base structure are perfectly bonded.

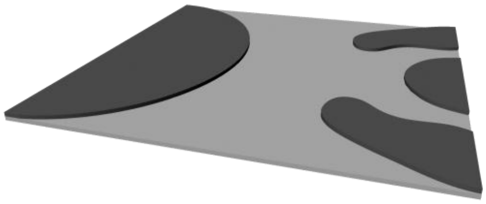

Fig. 1 A structure with surface damping layer

We are interested in finding the optimal distribution of a given amount of damping material within a prescribed design domain for minimizing the vibration amplitudes at specified positions. The topology optimization problem is thus formulated as

$$\begin{aligned}
&\min_{\boldsymbol{\rho}} \quad f=\sum_{j=1}^{m}A_j^2 \\
&\text{s.t.} \quad (-\theta^2\mathbf{M}+i\theta\mathbf{C}+\mathbf{K})\mathbf{Y}=\mathbf{F}, \\
&\qquad \sum_{e=1}^{N_e}\rho_e V_e^0 - f_{\mathrm{v}}\sum_{e=1}^{N_e}V_e^0 \le 0, \\
&\qquad 0<\rho_{\min}\le\rho_e\le 1 \ \left(e=1,\dots,N_e\right).
\end{aligned} \tag{1}$$

Analogously as in the elastic constant interpolation used in the SIMP model, an artificial damping model is suggested here for penalizing intermediate densities. Thus the elemental damping matrix is assumed as $\mathbf{C}_e^{\mathrm{d}}=\alpha_0^{\mathrm{d}}\left(\rho_e\right)^{q_1}\tilde{\mathbf{M}}_e^{\mathrm{d}}+\beta_0^{\mathrm{d}}\left(\rho_e\right)^{q_2}\tilde{\mathbf{K}}_e^{\mathrm{d}}$.

RESPONSE AND SENSITIVITY ANALYSIS

Response analysis

The governing equations for the finite element model of a structure with bonded damping layer patches under external harmonic excitations can be rewritten in terms of the state vector as

$$\mathbf{H}\dot{\mathbf{z}}(t)+\mathbf{G}\mathbf{z}(t)=\hat{\mathbf{f}}(t), \tag{2}$$

with

$$\mathbf{G}=\begin{bmatrix}\mathbf{K}^* & \mathbf{0}\\ \mathbf{0} & \mathbf{-M}^*\end{bmatrix},\ \mathbf{H}=\begin{bmatrix}\mathbf{C}^* & \mathbf{M}^*\\ \mathbf{M}^* & \mathbf{0}\end{bmatrix},\ \hat{\mathbf{f}}(t)=\begin{Bmatrix}\mathbf{f}^*\\ \mathbf{0}\end{Bmatrix}. \tag{3}$$

The generalized eigenvalue problem corresponding to Eq.(2) is $\left(\hat{\boldsymbol{\lambda}}\mathbf{H}+\mathbf{G}\right)\hat{\boldsymbol{\Phi}}=0$ .Using the complex mode superposition method, one can write the solution of Eq. (2) as $\mathbf{z}(t)=\hat{\boldsymbol{\Phi}}\mathbf{v}(t)=\sum_{k=1}^{2q}v_k\hat{\boldsymbol{\varphi}}_k$. By substituting $\mathbf{z}(t)$ into Eq. (2) and pre-multiplying it with $\boldsymbol{\Phi}^{\mathrm{T}}$, one can rewrite Eq. (2) as $\dot{v}_k-\hat{\lambda}_k v_k=\hat{f}_k^*(t)$.Thus the steady state displacement response of the vibrating structure $\mathbf{y}(t)$ can be obtained . Further, the vibration amplitude of the jth degree of freedom A_j can be calculated.

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Sensitivity analysis

A generalized structural behavior function $g(\mathbf{Y})$ only explicitly depending on the amplitudes of the displacement response $\mathbf{Y}$ is considered. The sensitivity analysis scheme for $g(\mathbf{Y})$ is derived by using the adjoint variable method (AVM). Considering the vibration equation and its conjugate equation, we can rewrite the function $g(\mathbf{Y})$ as

$$g = g(\mathbf{Y}) + \boldsymbol{\mu}_1^{\mathrm{T}}(\mathbf{WY} - \mathbf{F}) + \boldsymbol{\mu}_2^{\mathrm{T}}(\overline{\mathbf{W}}\overline{\mathbf{Y}} - \overline{\mathbf{F}}), \tag{4}$$

where $\overline{\mathbf{W}}$ and $\overline{\mathbf{F}}$ denote the conjugates of the dynamic stiffness matrix $\mathbf{W}$ and the force amplitude vector $\mathbf{F}$, respectively; $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$ are the adjoint vectors. Differentiating Eq. (4) with respect to the eth design variable leads to

$$\frac{\mathrm{d}g}{\mathrm{d}\rho_e} = \boldsymbol{\mu}_1^{\mathrm{T}}\frac{\partial \mathbf{W}}{\partial \rho_e}\mathbf{Y} + \boldsymbol{\mu}_2^{\mathrm{T}}\frac{\partial \overline{\mathbf{W}}}{\partial \rho_e}\overline{\mathbf{Y}} + \frac{\partial \mathbf{Y}^{\mathrm{R}}}{\partial \rho_e}\left(\frac{\partial g}{\partial \mathbf{Y}^{\mathrm{R}}} + \boldsymbol{\mu}_1^{\mathrm{T}}\mathbf{W} + \boldsymbol{\mu}_2^{\mathrm{T}}\overline{\mathbf{W}}\right) + \frac{\partial \mathbf{Y}^{\mathrm{I}}}{\partial \rho_e}\left(\frac{\partial g}{\partial \mathbf{Y}^{\mathrm{I}}} + i\,\boldsymbol{\mu}_1^{\mathrm{T}}\mathbf{W} - i\,\boldsymbol{\mu}_2^{\mathrm{T}}\overline{\mathbf{W}}\right). \tag{5}$$

Let the adjoint variables satisfy the following equations

$$\boldsymbol{\mu}_1^{\mathrm{T}}\mathbf{W} = \frac{1}{2}\left(-\frac{\partial g}{\partial \mathbf{Y}^{\mathrm{R}}} + i\frac{\partial g}{\partial \mathbf{Y}^{\mathrm{I}}}\right), \qquad \boldsymbol{\mu}_2^{\mathrm{T}}\overline{\mathbf{W}} = \frac{1}{2}\left(-\frac{\partial g}{\partial \mathbf{Y}^{\mathrm{R}}} - i\frac{\partial g}{\partial \mathbf{Y}^{\mathrm{I}}}\right). \tag{6}$$

It can be seen from Eq.(6) that $\boldsymbol{\mu}_1 = \overline{\boldsymbol{\mu}}_2$. Therefore it is sufficient to solve only $\boldsymbol{\mu}_1$ Eq. to determine the adjoint vectors in Eq.(5). Then Eq.(5) becomes

$$\frac{dg}{d\rho_e} = 2\,\mathrm{Re}\left(\boldsymbol{\mu}_1^{\mathrm{T}}\frac{\partial \mathbf{W}}{\partial \rho_e}\mathbf{Y}\right) = 2\,\mathrm{Re}\left(\boldsymbol{\mu}_1^{\mathrm{T}}\left(-\theta^2\frac{\partial \mathbf{M}}{\partial \rho_e} + i\,\theta\frac{\partial \mathbf{C}}{\partial \rho_e} + \frac{\partial \mathbf{K}}{\partial \rho_e}\right)\mathbf{Y}\right). \tag{7}$$

NUMERICAL EXAMPLE

In this example, the the optimal layout design of the damping layer attached to a cantilever square plate as shown in Fig.2 is considered. An external force $f(t) = Fe^{i\theta t}$ is applied at point I (the mid-point of the free edge), with $F = 10^5$ N, $\theta = 2\pi f_{\mathrm{p}}$ and f_{p}=30 Hz. the design domain is discretized by a 60×60 mesh. The volume fraction ratio of the damping material is restricted by $f_{\mathrm{v}} = 0.5$. The damping material distribution and the deformation contour of tthe optimal design are shown in Fig.3.

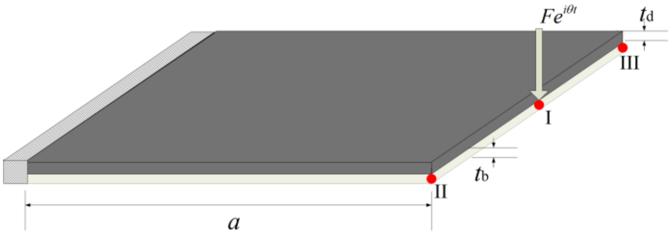

Fig. 2 A cantilever square plate with a time-harmonic load applied at the mid-point of the free edge

Fig. 3 Topology optimization design

CONCLUSIONS

This paper investigates topology optimization of damping layers in a vibrating structure under harmonic excitations. The structure is characterized by non-proportional damping model. The complex mode superposition method in conjunction with the state space approach, which can deal with the non-proportional damping, is employed to calculate the steady state response of the vibrating structure. The sensitivity analysis of the vibration amplitude is implemented by using the adjoint variable method. Numerical examples are given for illustrating the applicability and efficiency of the present approach. The influence of the number of eigenmodes involved in the response sensitivity analysis is discussed.

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