Particle swarm optimization in identification of stochastic multiscale models

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Summary The paper presents methodology of identification of material properties in multiscale modelling. The computational multiscale models contain stochastic material properties. The identification problem is solved as a minimization task with the use of particle swarm optimization.

INTRODUCTION

The paper is devoted to identification problems in multiscale modeling in stochastic conditions and extends previous works presented in [4]. The multiscale modelling incorporates two or more scales into analysis of a structure [5]. The numerical techniques which enable multiscale analysis of physical systems used in the paper is a computational homogenization. The identification allows one to evaluate materials or geometrical parameters of the structure in the microscale on the basis of measurements in the macroscale which have usually inaccurate character caused by uncertain nature of materials and geometrical parameters. One of the most important model of uncertainty used in identification problems is based on the theory of probability and stochastic processes [1]. The methodology presented in the paper takes into account stochastic nature of parameters in the microscale and the identification problem is formulated as minimization of a certain stochastic objective function. An approach based on the Particle Swarm Optimization (PSO) [2,3] is presented to the minimization problem.

PROBLEM FORMULATION

The goal of identification in multiscale modelling is to find a vector of material and geometrical parameters $\mathbf{X} = \left[X_i \right], i = 1, 2, .., n,$ treated as design variables on the micro-level which minimize an objective function dependent on state fields of displacements and strains on the macro-level of the structure [4]. In the considered problem the vector $\mathbf{X} = \mathbf{X}(\gamma), \gamma \in \Gamma,$ contains random parameters $X_i(\gamma)$ which should be determined from experimental statistical data where Γ is the space of elementary events which represents all the possible simplest outcomes of a trial associated with the given random phenomenon. One assumes that available measurement data of random displacements $\hat{\mathbf{u}}(\mathbf{x}_k, \gamma) \equiv \hat{\mathbf{u}}_k(\gamma)$ at sensor points $\mathbf{x}_k, k = 1, 2, ..., K,$ and strains $\hat{\varepsilon}(\mathbf{x}_l, \gamma) \equiv \hat{\varepsilon}_l(\gamma)$ at sensor points $\mathbf{x}_l, l = 1, 2, ..., L,$ are characterized by the mean values $m_{\hat{\mathbf{u}}_k} = \mathbf{E}\hat{\mathbf{u}}_k(\gamma)$ and $m_{\hat{\varepsilon}_l} = \mathbf{E}\hat{\varepsilon}_l(\gamma)$ and variances $\sigma^2_{\mathbf{u}_k} = \mathbf{E}\left[\hat{\mathbf{u}}_k(\gamma) - m_{\hat{\mathbf{u}}_k} \right]^2$ and $\sigma^2_{\hat{\varepsilon}_l} = \mathbf{E}\left[\hat{\varepsilon}_l(\gamma) - m_{\hat{\varepsilon}_1} \right]^2$, where $\mathbf{E}[\bullet]$ indicates expectation or ensemble average. The problem is formulated as minimization task a functional which represents a distance between measured $\hat{\mathbf{u}}_k(\gamma)$ and $\hat{\varepsilon}_l(\gamma)$ and theoretical values of displacements $\mathbf{u}(\mathbf{x}_k)$ and strains $\varepsilon(\mathbf{x}_l)$ computed for the numerical model:

$$\min_{\mathbf{X}(\gamma)} J \equiv a \sum_{k=1}^{K} \left| \mathbf{u}(\mathbf{x}_k) - \hat{\mathbf{u}}_k(\gamma) \right| + b \sum_{l}^{L} \left| \varepsilon(\mathbf{x}_l) - \hat{\varepsilon}_l(\gamma) \right| \tag{1}$$

This problem is converting into deterministic one (cf. [4]):

$$\min_{\mathbf{Y}} I \equiv a_1 \sum_{k=1}^{K} \left| \frac{m_{u_k} - m_{\hat{u}_k}}{m_{\hat{u}_k}} \right| + a_2 \sum_{k=1}^{K} \left| \frac{\sigma^2_{u_k} - \sigma^2_{\hat{u}_k}}{\sigma^2_{\hat{u}_k}} \right| + b_1 \sum_{l=1}^{L} \left| \frac{m_{\varepsilon_l} - m_{\hat{\varepsilon}_l}}{m_{\hat{\varepsilon}_l}} \right| + b_2 \sum_{l=1}^{L} \left| \frac{\sigma^2_{\varepsilon_l} - \sigma^2_{\hat{\varepsilon}_l}}{\sigma^2_{\hat{\varepsilon}_l}} \right| \tag{2}$$

where a new vector of design variables $\mathbf{Y}$ is formulated as $\mathbf{Y} = \left[(m_1, \sigma_1^2), (m_2, \sigma_2^2), ..., (m_i, \sigma_i^2), ..., (m_n, \sigma_n^2) \right]$ and a, b, a_1, a_2, b_1 and b_2 are scaling coefficients.

NUMERICAL EXAMPLE

As an example of identification of random parameters the two scale structure is considered (Fig. 1). The microstructure is built from two materials with stochastic parameters: Young's moduli $E_1(\gamma)$ and $E_2(\gamma)$ and Poison's ratios $\nu_1(\gamma)$ and $\nu_2(\gamma)$, respectively. The mean values of Young's moduli $m_{E_i}, i = 1, 2$ and Poisson's ratios $m_{\nu_i}, i = 1, 2$ are

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considered to be known and the Poisson's variances $\sigma^2_{\nu_i}, i=1,2$ are also known. The goal of the identification is to find variances of Young's moduli $E_1(\gamma)$ and $E_2(\gamma)$:

$$\mathbf{Y}=[\sigma^2_{E_1},\sigma^2_{E_2}] \tag{3}$$

with constraints $\sigma^{2-}_{E_i} \le \sigma^2_{E_i} \le \sigma^{2+}_{E_i}, i=1,2$ where limits of variances are given as follows: $\sigma^{2-}_{E_1}=0.0025\,[MPa]^2$; $\sigma^{2+}_{E_1}=2.2500\,[MPa]^2$ and $\sigma^{2-}_{E_2}=0.25\,[MPa]^2$; $\sigma^{2+}_{E_2}=225.00\,[MPa]^2$.

The known material properties are as follows: Young's modulus mean value m_{E_1} =3.4 [MPa], Young's modulus mean value m_{E_2} =72.0 [MPa], Poisson's ratio mean value m_{ν_1} =0.18, Poisson's ratio mean value m_{ν_2} =0.20, Poisson's ratio variance $\sigma^2_{\nu_1}$ =0.000625, Poisson's ratio variance $\sigma^2_{\nu_2}$ =0.000625.

The identification is performed on the basis of stochastic values of displacements for the macro model. The number of displacements sensor point was K=70 and displacements were measured in two directions on the lower part of the macromodel. The objective function described by (2) was applied for parameters $a_1=a_2=1$, $b_1=b_2=0$. The stochastic direct problem was solved by means of Monte Carlo FEM. The MSC.Nastran was applied for single FEM analysis in micro or macro-scales. The homogenization procedure was parallelized in presented approach.

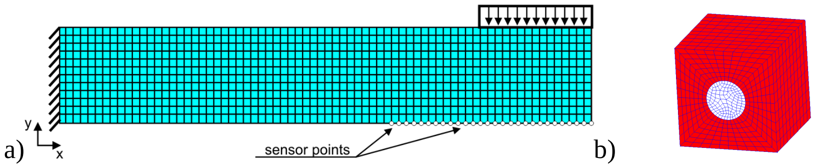

Figure 1. a) The macro model with sensor points and boundary conditions, b) the micromodel with two materials

The minimization task (2) with respect to **Y** (3) was performed with the use of the PSO with the following parameters: number of particles=20, best in swarm coefficient=1.44, best in neighbourhood coefficient=1.44, number of neighbours = 3, time step length= 0.3. The statistical measured displacements were numerically simulated for the testing purposes. The exact results were known before identification process. The results of identification are presented in Table 1. The obtained variances of Young's moduli are close to the actual ones. The change of properties of material number 2 had bigger influence on the displacements in the macroscale, the result obtained for this material are closer to actual one. The results obtained using PSO are similar to ones presented in [4] obtained with use of evolutionary algorithm.

Table 1. The results of identification

Parameter	Actual value	Obtained value
Young's modulus variance $\sigma^2_{E_1}$	0.25 [MPa]^2	0.14 [MPa]^2
Young's modulus variance $\sigma^2_{E_2}$	25.0 [MPa]^2	25.7 [MPa]^2

CONCLUSIONS

The methodology of identification of stochastic parameters of the micromodel on the base of statistical measurements performed for the macroscale with use of the particle swarm optimization was presented in the paper. The main advantage of the presented approach consists in the fact that a gradient of the objective functional is no needed and moreover there is a great probability of finding the global minimum. The numerical example of identification was presented and proved the proposed methodology.

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