GRADIENT BASED OPTIMIZATION OF PERIODIC LAYERED MATERIAL WITH SPECIFIC BAND-GAP CHARACTERISTICS

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Summary Periodic material can be elaborately designed to exhibit the capability of attenuating wave within certain frequency ranges, which are called stop bands or band gaps. Gradient-based optimization methods can not be directly used in the design optimization problem with the band-gap characteristics described by the attenuation coefficients of elastic waves, as the attenuation coefficient is not differentiable in frequency domain. Since the disperse relation of longitudinal elastic wave propagating in periodic layered materials can be analytically expressed by the cosine function of wave number, and the differentiable cosine functions can depict the tendency of attenuation coefficients in all the frequency domain, a direct gradient-based design optimization formulation with the band-gap characteristics depicted by the cosine function of wave number is investigated and developed. The objective functions including: (1) maximum the width of band gaps in desired frequency ranges; (2) maximum the attenuation efficiency of elastic wave in desired frequency ranges are achieved. The effectiveness of the optimization formulations are demonstrated by numerical examples.

Introduction

The periodic materials exhibit the capability of attenuating elastic wave within certain frequency range are called phononic band-gap materials [1]. The existence of elastic band gaps in periodic materials/structures may lead to many potential applications such as isolating devices, wave guides, acoustic mirrors, and acoustic lasers etc. Specific band gap characteristics including the width and location of band gaps can be achieved by artificial optimization design of the composite materials.

One of the optimization formulations of design band-gap materials with the goal of maximizing the relative band-gap between adjacent bands was achieved in the optimal design of two dimensional microstructure of phononic band gap material by topology optimization method [2].Similar optimization formulations has been used in optimization design of phononic band-gap materials including Mindlin plate [3], plane grid structures with non-structural masses [4], and etc. In this formulation of optimization design, Finite Element Method are used in the analysis of disperse relation and gradient-based optimization method such as MMA can be derived for analytical sensitivity. However, it is difficult in the optimization as the goal of the optimization is a (double) max-min objective with varying critical points. Another optimization formulation with the multiobjectives achieved by genetic algorithm was proposed in the design of phononic band-gap material [5]. Except for the formulations mentioned above, there are also some other optimization formulations for designing band gap materials.

In this paper, a simple gradient-based optimization formulation is proposed. The analytical relation between the cosine function of wave number and configurations of periodic layered material is derived. Then, utilizing the approximate relation between the cosine function of wave number and the attenuation coefficient of elastic wave, a direct gradient based design optimization formulation is investigated and developed for periodic layered material with specific band gap characteristics.

Disperse relation of longitudinal elastic wave propagation in periodic layered material

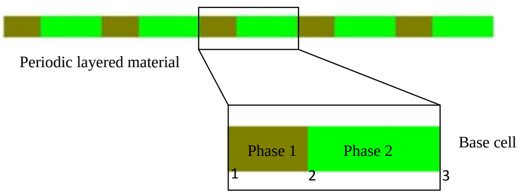

Fig. 1 Two-phase periodic layered material and the base cell

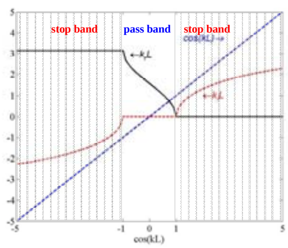

Fig. 2 Real and imagine parts of elastic wave number and cosine function of wave number

Based on the analytical solution of elastic wave propagation in periodic layered material, cosine function of wave number and the configurations of periodic layered material shown in Figure 1 can be written as:

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$$\cos(kL)=\left[\frac{1}{2}+\frac{1}{4}\left(\frac{\sqrt{E_1\rho_1}}{\sqrt{E_2\rho_2}}+\frac{\sqrt{E_2\rho_2}}{\sqrt{E_1\rho_1}}\right)\right]\cos\left[\omega\left(\frac{l_1}{\sqrt{E_1/\rho_1}}+\frac{l_2}{\sqrt{E_2/\rho_2}}\right)\right]+\left[\frac{1}{2}-\frac{1}{4}\left(\frac{\sqrt{E_1\rho_1}}{\sqrt{E_2\rho_2}}+\frac{\sqrt{E_2\rho_2}}{\sqrt{E_1\rho_1}}\right)\right]\cos\left[\omega\left(\frac{l_1}{\sqrt{E_1/\rho_1}}-\frac{l_2}{\sqrt{E_2/\rho_2}}\right)\right] \quad (1)$$

Since that, $\cos(kL)$ is smooth and differentiable as expressed in equation (1), and its variation tendency is the same with $k_i L$ in frequency domain, shown in Figure 2. Then the target band gap characteristics can be descript as indexes of integrations of $\cos(kL)$ within desired frequency ranges, such that gradient-based optimization methods can be carried out in optimization procedure.

Optimization Formulations

Model 1: maximizing the width of band gap in the desired frequency ranges.

Assume target frequency range is [1,3]kHz, and the two layers of materials are steal and rubber. Design variables are the thickness of each layer in the unit cell, and the constraints are the size of the unit cell. The formulation of optimization problem can be expressed as:

$$\begin{aligned}&\text{find}: && L=\{L_1,L_2\}^T\\&\max: && K_{\cos}=\int_{\omega_l}^{\omega_u}\cos^2(kL)\,d\omega\\&\text{s.t.}: && L_1+L_2\le L_0\end{aligned} \quad (2)$$

It can be concluded that relaxing size constraints will lead to the optimal design exhibits weaker attenuation capability from the optimal designs shown in Figure 3. Therefore, wave attenuation capacity of the medium should be measured by the attenuation quality of amplitude of displacement in unit length instead of that in unit cell. Simultaneously maximizing the width of band gap and maximizing the attenuation efficiency within desired frequency ranges offers new challenges in band-gap material design optimization.

Model 2: maximizing the attenuation efficiency of elastic waves in the desired frequency ranges.

Design variables and the material properties are the same with model 1. The formulation of optimization problem can be expressed as:

$$\begin{aligned}&\text{find}: && L=\{L_1,L_2\}^T\\&\max: && K=\int_{\omega_l}^{\omega_u}k_i^{\,2}\,d\omega\end{aligned} \quad (3)$$

Where attenuation efficiency is measured by the attenuation of elastic wave within unit length, expressed as k_i, which is:

$$k_i=\frac{1}{L}\mathrm{Re}\left[\log\left(\sqrt{\cos^2(kL)}+\sqrt{\cos^2(kL)-1}\right)\right] \quad (4)$$

The optimal design is 13.8mm for steel, 9.74mm for rubber, and the optimal size of the unit cell is 23.54mm. Figure 4 shows that in the same length (94.16mm, length of 4 optimal cells from model 2), the attenuation quality of optimal design of model 2 is larger than that of optimal design of model 1(a) in the desired frequency range [1,3]kHz.

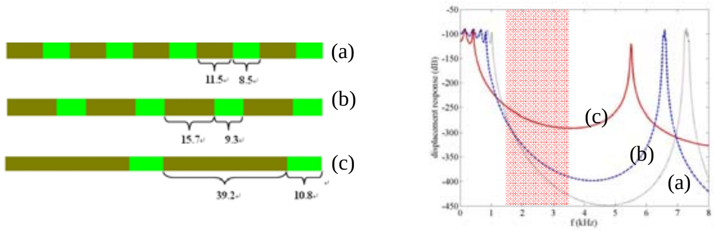

Fig.3 Left: Optimal designs subjected to size constraints as 20mm(a), 25mm(b) and 50mm(c) Right: corresponding frequency responses

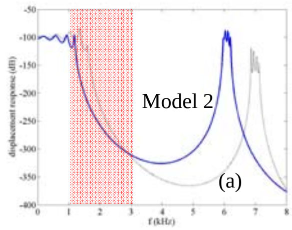

Fig.4 Compare of frequency responses of optimal design of model 2 and model 1(a)

Conclusions

A gradient based optimization formulation is proposed in the optimization design of periodic layered band-gap material by utilizing the cosine function of wave number, and the maximum width of band gap and maximum attenuation efficiency in desired frequency ranges has been achieved.

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