Isogeometric Shape Design Sensitivity Analysis of Elasticity Problems Using Multi-Resolution and *h*-Refinement Approach

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Summary An isogeometric shape DSA method is proposed using multi-resolution technique combined with *h*-refinement. In the multi-resolution method, to impose displacement compatibility, a transformation matrix relates the original and the updated sets in terms of control points and weights of NURBS basis function. Even though enforcing C^0 continuity at interfaces, the compatibility does not have a significant impact on the response at the region sufficiently far away from the interface so that sufficient accuracy can be obtained at the region of interest through the multi-resolution technique. To obtain efficient and precise adjoint shape sensitivity, correct normal and curvature information should be taken into account. However, in conventional finite element methods using linear interpolation functions, the normal and curvature are inaccurate or missing due to the lack of inter-element continuity of design space. In this paper, an accurate and efficient shape DSA method is developed using the multi-resolution approach.

KEY WORDS: Isogeometric Method, Shape Design Sensitivity, Multi-Resolution, *h*-Refinement, Oslo Algorithm

Extended Abstract

In this paper, for general elasticity problems, an isogeometric shape design sensitivity analysis (DSA) method is proposed using a multi-resolution technique combined with *h*-refinement strategy. In an isogeometric framework, the NURBS basis functions that represent the geometry in CAD systems can be directly used in both response and sensitivity analyses. The NURBS basis functions enable to incorporate exact geometry into the framework of response analysis by discretizing the domain of analysis at the early stage of geometry definition. When knot vectors are given, the NURBS basis functions are easily obtained recursively and very simple to enhance the order of basis functions. Mesh refinement strategy in the isogeometric approach includes the classical *h*- and *p*-refinements and the new possibility of *k*-refinement. The refinements are easily implemented to maintain the exact geometry without subsequent communication with the CAD systems.

The isogeometric model can be refined by inserting a number of knots for the NURBS without altering the given geometry. In the knot refinement process of Oslo algorithm, depending only on the order of basis function, multiple knots are inserted at once. Further refined geometry can be obtained to offer better quality of responses by repeating the knot insertion. However, due to the tensor product nature of NURBS, the *h*-refined knot vectors result in many of unwanted degrees of freedom, which also could introduce unwanted ripples in the surface. Thus, a local refinement scheme for the sub-domain patches of multi-resolution is inevitable. To prevent the possible discontinuity of responses between different resolution sub-domains, a displacement compatibility condition is enforced using the *h*-refinement capability. In the multi-resolution method, to impose the displacement compatibility condition, the transformation matrix relates the original and the updated sets in terms of the control points and the weights of NURBS basis function. Since the compatibility condition has only C^0 continuity at the interface, a stress contour could show discontinuity across the sub-domains.

Using the multi-resolution approach, the isogeometric analysis method is applied to the stress concentration problem of infinite plate with a hole in Figure 1, where local refinements near highly stressed region are necessary. Even though enforcing C^0 continuity, the compatibility does not have a significant impact on the response at the region sufficiently far away from the interface so that the sufficient degrees of accuracy can be obtained at the region of interest through the multi-resolution technique. For the given traction of 10 in Figure 1, the stress in two patch multi-resolution model (locally refined) converges faster than that in single patch model (uniformly refined).

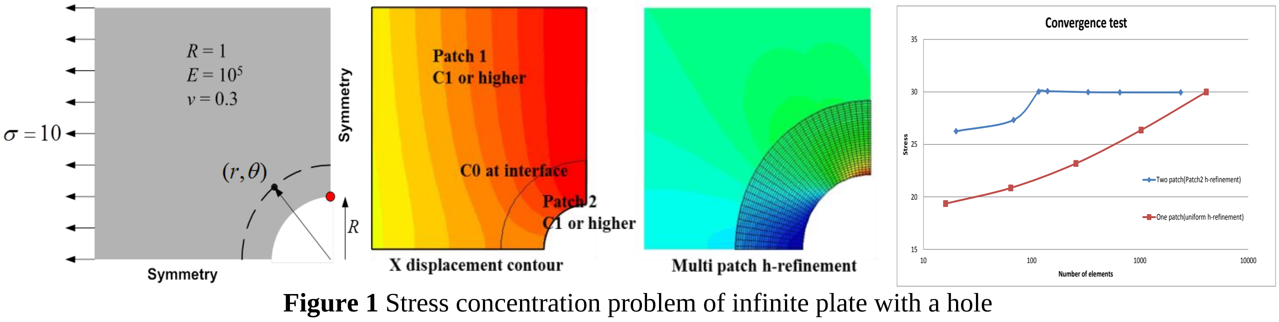
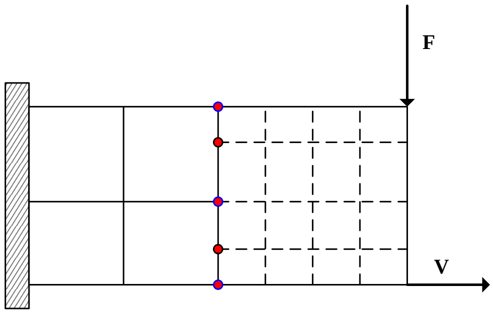


Figure 1 Stress concentration problem of infinite plate with a hole

To obtain efficient and precise adjoint shape sensitivity, correct normal and curvature information should be taken into account in shape sensitivity expressions. However, in conventional finite element methods using linear interpolation functions, the normal and curvature information is generally inaccurate or missing due to lack of inter-element continuity of design space. In this paper, a continuum-based shape DSA method is developed using the multi-resolution approach. Also, the accurate sensitivity can be utilized to obtain an optimal design in a very efficient way. The sensitivity results are compared with analytical solutions and finite difference sensitivity to demonstrate the feasibility and accuracy of the proposed method. For the shape variation of cantilever problem in Figure 2, the right bottom point is perturbed in the direction of V. The analytical displacement sensitivity (DDM) along the interface is compared with finite difference sensitivity (FDM), which shows excellent agreements at all the degrees of freedom.



(a) Physical model

Degree of Freedom	FDM	DDM	Agreement (%)
27	2.119901E-16	2.119858E-16	99.9980
28	-5.862352E-13	-5.862352E-13	100.0000
29	-1.202520E-16	-1.202536E-16	100.6017
30	1.061280E-15	1.061262E-15	101.7446
31	5.633344E-17	5.633431E-17	100.0010
32	1.779400E-16	1.779280E-16	100.0037
33	3.009805E-17	3.010130E-17	99.9990
34	2.485500E-16	2.485355E-16	99.9971
35	-1.996402E-16	-1.996324E-16	100.0001
36	1.857899E-16	1.857702E-16	100.0010

(b) Shape sensitivity

Figure 2 Cantilever problem

Acknowledgements

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (Grant Number 2010-18282). The support is gratefully acknowledged.

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