MULTI-MATERIAL TOPOLOGY DESIGN OF LAMINATES WITH STRENGTH CRITERIA

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Summary The objective of this paper is to present a novel approach for multi-material topology optimization of laminated composite structures where strength constraints are taken into account together with other global structural performance measures. The topology design problem considered contains very many design variables, and when strength criteria are included in the problem, a very large number of criteria functions must be considered in the optimization problem to be solved. Thus, block aggregation methods are introduced, such that global strength measures are obtained. These formulations are illustrated for multi-material laminated design problems where the maximum failure index is minimized while compliance and mass constraints are taken into account.

INTRODUCTION

Laminated composite structures are being used in many different engineering applications due to their superior strength and stiffness characteristics. In order to fully exploit the weight saving potential of these multilayered structures, it is necessary to tailor the laminate layup and behavior to the given structural needs. In many applications the glass or carbon fiber reinforced polymers (FRP) are combined with core materials (foam, balsa tree, etc.) in parts of the structure in order to introduce sandwich structures, which may yield better structural performance for a given cost than monolithic fiber-reinforced polymer structures. In this work the combinatorial problem of proper choice of material and fiber orientation is formulated as a material selection problem using the so-called Discrete Material Optimization (DMO) approach developed by the authors [1]. This approach relaxes the discrete material selection problem to a continuous formulation using interpolation schemes with penalization, i.e., the material properties are computed as weighted sums of properties of candidate materials, which may be different kinds of fiber reinforced materials associated with given fiber angles together with core materials. The DMO approach has been used for global measures such as stiffness, mass, cost, eigenfrequencies, and buckling load factors of linear and geometrically nonlinear problems, and in this work local measures of performance in terms of strength criteria are included.

PARAMETERIZATION

The approach relies on newly developed interpolation schemes, see [2] for compliance problems, that are generally applicable for interpolation between an arbitrary number of pre-defined candidate materials. Here the "generalized SIMP scheme" is used where the material property of interest, for example the constitutive matrix $\mathbf{C}^{eff}$, is computed in the following way, when there are n_c candidate materials to choose between, each characterized by its constitutive matrix $\mathbf{C}_i$:

$$\mathbf{C}^{eff} \;=\; \sum_{i=1}^{n^c} x_i^p \mathbf{C}_i, \quad 0 \le \underline{x}_i \le x_i \le \overline{x}_i \le 1, \quad \sum_{i=1}^{n^c} x_i = 1 \tag{1}$$

Thus, design variables x_i are directly associated with candidate material i, and the penalization power p is used to enforce a unique choice of material at the end of the optimization. With this parameterization a design variable x_i represents the volume fraction of the associated candidate material.

The parameterization can be applied element-wise for each layer in the finite element model consisting of layered shell finite elements, but typically larger patches of elements, associated with the same parameterization, are used. A large number of sparse linear constraints to enforce the selection of at most one material in each design domain is introduced, but these constraints are handled effectively using the SNOPT optimization package [3] using either SLP or SQP.

STRENGTH CRITERIA AND BLOCK AGGREGATION METHODS

The inclusion of strength criteria introduces a large number of highly nonlinear (local) constraints which in combination with the many design variables introduced by the DMO approach yield a computationally challenging optimization problem. The maximum strain failure criterion is used for the example in this paper, but many other failure criteria used for laminated composites have also been implemented and can be applied. The failure criteria used for FRP materials are normally defined in the material coordinate system 1-2-3, and the procedure applied for computing effective failure indices (FI_{eff}) with the above DMO parameterization is outlined in the following. First the structural problem is solved using Eq. 1 for determining the effective constitutive matrices according to the current values of the design variables. In the failure analysis the strain vector $\boldsymbol{\epsilon}$ in the structural coordinate system is computed for top and bottom position of each layer in a given finite element. For each candidate material i the strain vector $\boldsymbol{\epsilon}$ is transformed to the material coordinate system 1-2-3 of the candidate material, resulting in a strain vector $\boldsymbol{\epsilon}_i^{1-2-3}$, and then

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the failure index $\mathrm{FI}_i(\boldsymbol{\epsilon}_i^{1-2-3})$ in the material coordinate system is evaluated. Based on this the effective failure index FI_{eff} (ElemNo, LayerNo, top/bottom) $= \sum_{i=1}^{n^c} x_i{}^q \, \mathrm{FI}_i(\boldsymbol{\epsilon}_i^{1-2-3})$ is computed, i.e. if the power $q = 1$ is chosen, a linear weighting according to the volume fractions of the candidate materials is used.

The inclusion of strength criteria introduces a large number of criterion values (number of elements $\times$ number of layers per element $\times$ 2). In order to reduce the amount of functions to include in the optimization problem, globalization functions are used. In this work global strength measures (see, e.g., [4, 5, 6]) are obtained by using Kreisselmeier-Steinhauser (KS) functions, but other globalization functions like the p-norm are also being investigated.

The efficiency of using global strength approaches decreases when a large number of values are lumped into a single global value, but this problem can be handled by associating a global strength measure with each patch used for the parameterization. The approach has similarities with the block aggregation approach [6] and the regional stress measure approach [5] used for single-material structural topology optimization problems with stress constraints. The number of FI values, n^{FI}, to include for each patch may be defined by the user, and for each patch j, $j = 1, \ldots, n^P$, a KS function $f(\mathbf{x})^j$ is computed:

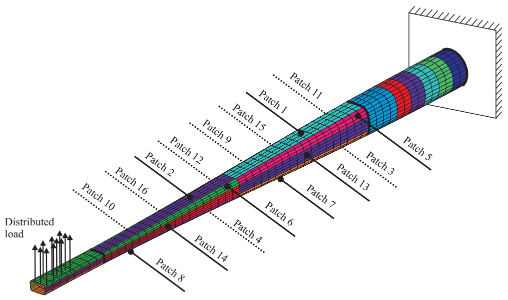
$$f(\mathbf{x})^j \;=\; \frac{1}{\rho} \ln \left[\sum_{k=1}^{n^{FI}} e^{\rho \, f_k^j} \right] \quad \text{or} \quad f(\mathbf{x})^j \;=\; f^{j,max} + \frac{1}{\rho} \ln \left[\sum_{k=1}^{n^{FI}} e^{\rho \, (f_k^j - f^{j,max})} \right] \tag{2}$$

where $\ln = \log_e$, the scalar ρ typically is between 2 and 200 (50 is used), and $f^{j,max}$ is the largest FI value among $f_k^j(\mathbf{x})\,,\; k = 1, \ldots, n^{FI}$. The latter KS definition has numerical advantages.

The optimization problems are solved using mathematical programming. Strength criteria can be included as objective function or as constraints, and in both cases the optimization problem must be solved by an algorithm that in an efficient way can handle the many linear constraints ($\sum x_i = 1$) generated by the DMO parameterization scheme used. The examples considered have been solved using a SLP approach where SNOPT [3] is used to solve the LP sub-problems. A bound formulation is applied for solving the min-max problems obtained when strength is used as objective.

EXAMPLE AND CONCLUDING REMARKS

The approach is shortly demonstrated in the following for multi-material design of a generic main spar from a wind turbine blade subjected to the maximum flapwise bending load case. The midsection of the main spar is divided into 16 patches, each consisting of 10 layers of equal thickness. The candidate materials are GFRP oriented at $0°$, $45°$, $-45°$, and $90°$, GFRP $\pm 45°$ biax mats, and foam material. The objective has been to minimize the failure index while fulfilling compliance and mass constraints. 1/5 of the design domain should be filled with foam material. The FE model and the parameterization together with the DMO results are seen on Figure 1. Both constraints are active for the final design.



Patch \ Layer	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10
P1	$90°$	$0°$	$0°$	$0°$	$0°$	$0°$	$0°$	$0°$	$0°$	$0°$
P2	$90°$	$0°$	$0°$	$0°$	$0°$	$0°$	$0°$	$0°$	$0°$	$0°$
P3	$0°$	$0°$	$0°$	$0°$	$0°$	$0°$ / $90°$	$0°$	$0°$	$0°$	$0°$
P4	$0°$	$0°$	$0°$	$0°$	$0°$	$0°$	$0°$	$0°$	$0°$	$0°$
P5	$90°$	foam	foam	foam	$0°$	$0°$ *	$0°$	$0°$ *	$0°$	$0°$
P6	$-45°$	$45°$	$45°$	$90°$	$0°$	$0°$	$0°$	$0°$	$0°$	$45°$
P7	$45°$ *	$45°$ *	foam	$45°$ *	$0°$	$0°$	$0°$	$0°$	$0°$	$0°$
P8	$-45°$	$0°$	$0°$	$0°$	$0°$	$0°$	$0°$	$0°$	$0°$	$0°$
P9	$90°$ *	foam *	foam	foam	foam	foam	foam	$-45°$ *	$-45°$ *	$0°$
P10	$45°$	$45°$	$0°$	$0°$	$0°$	$0°$	$0°$	$0°$	$0°$	$45°$
P11	$90°$	$90°$ *	$90°$ *	$0°$ *	$0°$	$0°$	$0°$	$45°$	$45°$ *	$0°$
P12	$90°$	$-45°$	$-45°$	$90°$	foam	foam	$0°$	$0°$	$-45°$	$-45°$
P13	$90°$	foam	foam	foam	foam	foam	foam	foam	foam	$0°$
P14	$0°$	$0°$	$0°$	$0°$	$0°$	$0°$	$0°$	$0°$	$45°$	$45°$
P15	$90°$	foam	foam	foam	foam	foam	foam	foam	foam	$0°$
P16	$0°$	$0°$	$0°$	$0°$	$0°$	$0°$	$0°$	$0°$	$0°$	biax45

Figure 1. Left: Generic main spar from wind turbine blade. Right: Results of optimization. The candidate material associated with the largest design variable is listed in the table for each layer in each of the 16 patches.

In general, a distinct choice of material is obtained in most of the design domain, and the presented approach is a step towards efficient handling of local failure criteria in topology design of multi-material laminated composite structures.

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