STRUCTURAL TOPOLOGY OPTIMIZATION USING MESHFREE METHODS

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Summary This paper aims to propose a new structural topology optimization method using a dual-level point-wise density approximant and the meshless Galerkin weak-forms, totally based on a set of arbitrarily scattered field nodes in the design domain. One benchmark numerical example is employed to demonstrate the effectiveness of the proposed design method.

INTRODUCTION

Structural topology optimization has experienced considerable development with many successful applications in a wide range of engineering disciplines over the past two decades [1]. Topology optimization essentially consists of a numerical procedure to iteratively redistribute a given amount of material in the fixed extended design domain subjecting to supports and loads, in order to determine a best material layout to optimize the design objective function under specific constraints. So far several typical methods have been developed for topology optimization [2]. One objective of this research is to develop a dual-level point-wise density approximant to describe the relationship between material densities and the practical material properties, in terms of the key concept of conventional SIMP. The generic design variables and nodal density variables are uniformly described by a set of scattered field nodes in the design domain. Firstly, the "CS-Shepard function" is included in the problem formulation to enhance the smoothness of nodal density field. Secondly, the meshless "MLS-Shape function" is applied to approximate another point-wise density field for the computational points independent of field nodes. Finally, the dual-level point-wise density approximant with the Galerkin global weak-forms are used to implement the meshless approximation of state discrete equations. Hence, this paper aims to propose a real point-wise density variables-based method for topology optimization, totally according to a set of arbitrarily scattered nodes.

1. APPROXIMATIONS USING "CS-SHEPARD FUNCTION"

The moving least squares (MLS) technique is used to construct the meshless shape function [3]. One major advantage of MLS meshless approximation is it can inherit the continuity of the weight function. A lower order polynomial basis can be used to generate higher continuous approximations by choosing a proper weight function, and so the weight function plays an important role in meshless approximations. The MLS-Shape function of the first-order consistency lacks of the Kronecker delta property. MLS shape functions will degenerate to a special family of functions - the Shepard function, which owns basic properties which are particularly useful for a physically meaningful approximate with the cubic spine (CS) weight function The special Shepard function is termed as "CS-Shepard function".

This section is concerned with a nodal density-based interpolation scheme via the CS-Shepard function, based on the key concept of conventional elemental SIMP methods. The proposed point-wise SIMP in this study is illustrated as a density-stiffness interpolation that establishes a dependency between nodal material properties and nodal densities only in terms of a set of arbitrarily scattered field nodes. Based on the above discussion, the Shepard functions is defined by

$$\Phi_I(x) = \tilde{w}_I(x) \Big/ \left(\sum_{J=1}^{N_s} \tilde{w}_J(x) \right) = \tilde{w}(x - x_I) \Big/ \left(\sum_{J=1}^{N_s} \tilde{w}(x - x_J) \right) \quad (I = 1, ..., N_s) \tag{1}$$

In this "CS-Shepard function", many functions can act as its weight function provided that they satisfy the basic conditions [47], such as compactly supported, continuity and normality, and non-negative over the local domain. The commonly used weight functions include the exponential, the cubic spline and the quartic spline functions [3,4]. This work employs a singular cubic spline (CS) weight function with compactly supported influence domain. The singular weight function possesses the non-negative and range-bounded properties, and normality (unity) property, so which can be used to ensure a physical meaningful density field interpolation in topology optimization.

2. A NODAL DENSITY-BASED INTERPOLATION SCHEME

The aforementioned "CS-Shepard function" is then used to construct a nodal density-based interpolation scheme. Inside the design domain, the density at any node can be expressed as follows in terms of the related nodal density variables:

$$\rho(x_I) = \sum_{I=1}^{N_s} \Phi_I(x_I) \rho_I = \sum_{I=1}^{N_s} \left(\rho_I \tilde{w}_I(x_I) \Big/ \sum_{J=1}^{N_s} \tilde{w}_J(x_I) \right) \tag{2}$$

It is straightforward that the CS-Shepard function can meet the necessary conditions to make a physically meaningful density interpolation for topology optimization. This density interpolation is included in the problem mathematical formulation. So, the material Young's modulus at any node can be defined by

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$$E(x_I) = \rho^p(x_I)E_0 = \left(\sum_{I=1}^{N_s} \Phi_I(x_I)\rho_I\right)^p E_0 \tag{3}$$

3. FORMULATION USING MESHLESS GALERKIN WEAK-FORMS

In this study, the linear elastic structures are considered only for the sake of numerical simplicity but without losing any generality. It is necessary to develop the weak form corresponding to the equilibrium equations and the boundary conditions in order to get the discrete equations. In terms of the integration by parts and the divergence theorem, the final discrete equations corresponding to the weak formulation can be obtained [1,3,4]. In numerical implementation, the Gauss quadrature [3] is applied to calculate the system stiffness matrix by looping the virtual background cells by

$$\mathbf{K}=\sum_{ii=1}^{4}\sum_{jj=1}^{4}\left\{\mathbf{B}^{\mathrm{T}}(s_{ii},t_{jj})\left(\rho^{p}(s_{ii},t_{jj})\mathbf{D}_{0}\right)\mathbf{B}(s_{ii},t_{jj})\left|\mathbf{J}(s_{ii},t_{jj})\right|w_{ii}w_{jj}\right\} \tag{4}$$

where the point-wise density at any computational point can then be approximated as follows:

$$\rho(s_{ii},t_{jj}) = \sum_{J=1}^{n_s} N_J(s_{ii},t_{jj})\rho(x_J)\text{, where } \rho(x_J) = \left(\sum_{I=1}^{N_s}\Phi_I(x_I)\rho_I\right) \tag{5}$$

4. OPTIMIZATION FORMULATION

In this study, the structural mean compliance design is applied to demonstrate the effectiveness of the proposed topology optimization method. The mean compliance problems [1] can be mathematically defined as follows:

$$\begin{cases} \underset{\rho_i\,(i=1,2,\ldots,N_e)}{\text{Minimize}}: J(u) = \int_\Omega f(u,\delta u)d\Omega \\ \text{Subject to:} \begin{cases} \int_\Omega \rho_i V_i d\Omega - V_0 \le 0 \\ 0 < \rho_{\min} \le \rho_i \le 1 \\ \mathbf{a}(u,\delta u) = \mathbf{l}(\delta u),\ \ u\big|_{\Gamma_D} = \overline{u}, \forall \delta u \in \mathbf{H}^1 \end{cases} \end{cases} \tag{6}$$

The adjoint sensitivity method (ASM) is applied to perform the design sensitivity analysis. We can find the design sensitivity of the objective function with respect to the design variables simply as

$$\frac{\partial J(\rho_i)}{d\rho_i} = -\mathbf{u}^{\mathrm{T}}\frac{\partial \mathbf{K}}{\partial \rho_i}\mathbf{u}\text{, where } \frac{\partial \mathbf{K}}{\partial \rho_i} = \sum_{ii=1}^{4}\sum_{jj=1}^{4}\left\{\mathbf{B}^{\mathrm{T}}(s_{ii},t_{jj})p\left(\rho(s_{ii},t_{jj})\right)^{p-1}\mathbf{D}_0\mathbf{B}(s_{ii},t_{jj})\left|\mathbf{J}(s_{ii},t_{jj})\right|w_{ii}w_{jj}\right\} \tag{7}$$

After finding the design sensitivities, many efficient gradient-based optimization algorithms can be used as the optimizer to find the solution, such as the OC and the MMA approaches [1].

5. NUMERICAL EXAMPLE

The benchmark example [1,2], MBB (miniature bending beam) structure (Fig. 1), is used as the numerical cases, to which a vertically force F=5000 is loaded at the center of the upper side of the structure. The bottom-left corner is fixed and the bottom-right corner is simply supported as a roller. The allowed material usage is limited to 50%.

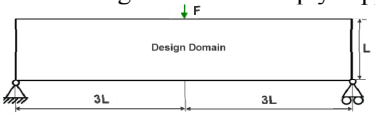

Fig. 1. Design domain of the MBB

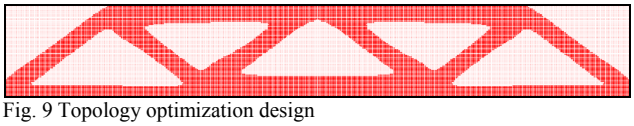
Fig. 9 Topology optimization design

The design domain is discretized with 31×91=2821 field nodes. The optimization is converged after 73 iterations, and structural mean compliance is minimized from 849.3539 to 210.1906. The proposed method can get the optimal design (Fig. 2) with a small number of iterations, and the volume constraint is also mass conservative.

6. CONCLUSIONS

This study proposes a new topology optimization method for continuum structures using a dual-level point-wise density approximant and the MLS meshless Galerkin method. One benchmark numerical example for the structural mean compliance is used to demonstrate the effectiveness of the proposed method. It is straightforward to extend the proposed method to more advanced topology optimization problems.

References

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