SIMULATION AND OPTIMIZATION OF THIN CORRUGATED MEMBRANES

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Summary In the paper, axisymmetric round membranes with corrugated surface under hydrostatic pressure are considered. To simulate the elastic behavior the non-linear equations based on the Kirchhoff-Love hypotheses are used. With the help of genetic algorithm a number of optimal profile design problems for such membranes are solved.

INTRODUCTION

Designing of thin-walled structures is increasingly difficult as they decrease in size and increase in complexity, new materials are used for their construction. The rate of change is increasing, often due to forces of globalization, new technological capabilities, rising consumer expectations. One of the most formidable problems in designing is closely connected with tightening requirements to the work of construction. Predictability and reliability of construction behavior under extreme load conditions, complex stress states, long-term operation, influence of radiation, high and low temperatures are extremely attractive performance characteristics for engineers. Purely technical approach to analysis and design of such advanced thin-walled constructions often lead to failure, as a more comprehensive approach based on development of new models that can accurately describe the nonlinear behavior of membranes and efficient optimization methods is required. All these show the urgency of the membranes form-finding problems presented in this paper.

This paper presents solid background on approach used to simulate membranes behavior and brief illustrations of some optimization problems devoted to the search of optimal corrugation forms.

MEMBRANE BEHAVIOR SIMULATION

Let consider axisymmetric round membrane of the thickness h and radius $\rho >> h$ under applied hydrostatic pressure. The surface profile is denoted by the smooth function $z = f(r)$, where z – characteristic of the shape that describe the dependence of the middle surface prominence on the membrane radius. Principal surface curvatures along meridian and parallels are defined by

$$k_1 = -\frac{f''}{(1+f'^2)^{\frac{3}{2}}}, \quad k_2 = -\frac{f'}{r(1+f'^2)^{\frac{1}{2}}}, \quad f' = \frac{df}{dr}.$$

In the case of axisymmetric deformation of the membrane the non-linear equilibrium equations based on the Kirchhoff-Love hypotheses [1] can be reduced to the differential equations system of first order regarding dimensionless components of vector **Y**:

$$Y_1 = \frac{u_1}{h}, \quad Y_2 = \frac{u_2}{h}, \quad Y_3 = \theta_1, \quad Y_4 = \frac{rT_{11}}{Bh}, \quad Y_5 = \frac{r(CQ_1 - T_{11}\theta_1)}{Bh}, \quad Y_6 = \frac{rM_{11}}{Bh^2}, \tag{1}$$

where u_1, u_2 are the displacements of the middle surface points along the meridian and parallel respectively, θ_1 is angle of normal rotation, T_{11} and M_{11} are axial force and bending moment. The system obtained is expressed as

$$\begin{gathered}
\frac{dY_1}{dx} = -\frac{\nu}{x}Y_1 - \frac{K_1+\nu K_2}{C}Y_2 + \frac{1}{Cx}Y_4 - \frac{1}{2C}Y_3^2, \qquad \frac{dY_2}{dx} = -\frac{K_1}{C}Y_1 - \frac{1}{C}Y_3, \\
\frac{dY_3}{dx} = -\frac{\nu}{x}Y_3 + \frac{12}{Cx}Y_5, \qquad \frac{dY_4}{dx} = \frac{(1-\nu^2)C}{x}Y_1 - (1-\nu^2)K_2Y_2 + \frac{px}{C}Y_3 + \frac{\nu}{x}Y_4 - \frac{K_1}{C}Y_6, \\
\frac{dY_5}{dx} = \frac{(1-\nu^2)C}{12x}Y_3 + \frac{1}{C}Y_6 - \frac{\nu}{x}Y_5 + \frac{1}{C}Y_3Y_4, \\
\frac{dY_6}{dx} = (1-\nu^2)K_2Y_1 + \frac{(1-\nu^2)K_2^2x}{C}Y_2 + \frac{K_1+\nu^2K_2^2}{C}Y_4 - \frac{px}{2C}Y_3^2 + \frac{px}{C}.
\end{gathered} \tag{2}$$

Here $K_i = k_ih$ are dimensionless curvatures, $x = r/h$ is dimensionless radius, $C = C(r) = (1+f'^2)^{-1/2}$. Boundary conditions for (2) at $x_1 = \rho/h$ in the case of rigidly fixed boundary:

$$Y_1(x_1) = 0, \quad Y_2(x_1) = 0, \quad Y_3(x_1) = 0. \tag{3}$$

For the numerical integration of system (2) the approach replacing boundedness of solution at $x = 0$ by boundary conditions at $x = x_0$ ($x_0 \leq 1$) was used:

$$Y_4(x_0) = 0, \quad Y_5(x_0) = 0, \quad Y_6(x_0) = 0. \tag{4}$$

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Numerical calculation for the nonlinear two-point boundary problem (2)-(4) can effectively be done with the shooting method, in which loading parameter is a value of vertical deflection at $x = x_0$, while the set of other shooting parameters $V_1 = Y_1(x_0)$, $V_2 = p$ and $V_3 = Y_3(x_0)$ is determined after solving Cauchy problem.
This problem has the stable and durable solution, which can be found in an acceptable time and help to obtain all necessary data about membrane stress state. Using this information some characteristics of the membrane can be calculated for futher optimization process. As an example of such characteristic, function of surface stresses can be considered, as smoothing of this function lead to decrease of plastic deformation zones, cause of thin shells fracture. In this work, the function of volume change under live pressure is suggested as a characteristic to optimize.

NUMERICAL SHAPE OPTIMIZATION

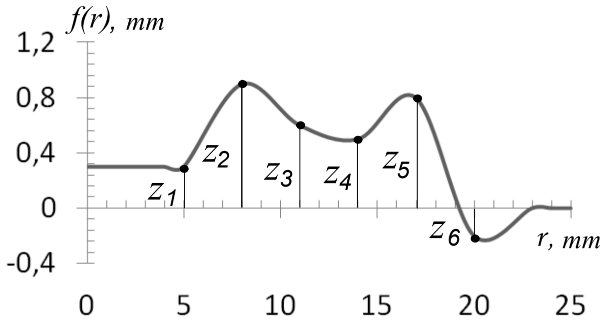

Figure 1. Membrane profile: varying heights z_i.

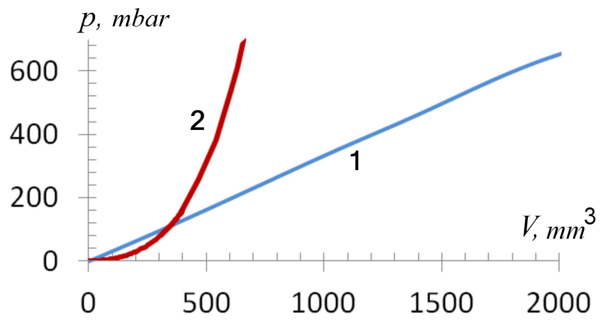

Figure 2. Membrane optimization result.

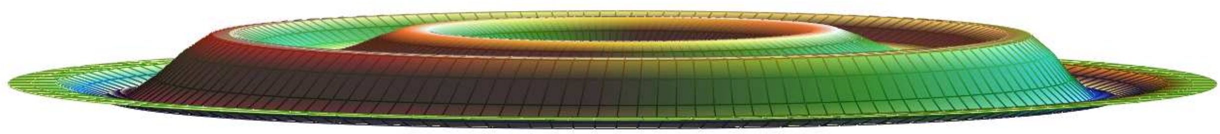

Figure 3. 3-D image of the optimal membrane.

Corrugation can decrease stresses in membrane, make it more flexible or, on the contrary, ensure necessary rigidity. Therefore, reasonable profile design can considerably enhance membrane performance. During this research, a number of optimization problems with different optimal criteria was solved. In fact, some of the optimal membranes were produced and involved into engineering projects. It was decided that the best method to adopt for this optimization problems was modified genetic algorithm [2], a programming technique that mimics biological evolution as a problem-solving strategy.
Common optimization problem for this work was search of the membrane characteristics (the dependence of the applied pressure on the change of the volume under the membrane surface) with linear, parabolic or exponential behavior. Another desired factor in the performance of membrane was flatness of its characteristic.
There have been several investigation into optimization of membranes already in existence [2] – slight varying of some geometric parameters of the fixed profiles led to necessary changes in the characteristic behavior. Up to now, the research has tended to focus on free-form membranes (flat enough), as they give much more promising results for practical application.
As a demonstration let consider membrane profile that can be described by heights z_i, $(i = 1..6)$ in the fixed radial points r_i, $(i = 1..6)$. In an attempt to find the membrane with the highest possible extensive linear zone (optimization problem), z_i are varied in the fixed interval (Fig. 1). The most striking result for fixed accuracy is illustrated on figures above: optimal membrane profile (Fig. 1), its highest possible extensive linear characteristic (Fig. 2) and 3-D image (Fig. 3). To emphasize benefits of such corrugation the load distribution lines of the optimal membrane(Fig. 2-1) and plate of the same diameter (Fig. 2-2) are compared.

References

[1] Getman, I. P., Karyakin, M. I., Ustinov, U. A.: Analysis of the non-linear behavior of circular membranes with an arbitrary radial profle. *J. Appl. Math. Mech.* **74(6)**:654–662, 2010.
[2] Karyakin M., Sigaeva T.: Application of Genetic Algorithms to the Shape Optimization of the Nonlinearly Elastic Corrugated Membranes. *Adv. Srtuct. Materials* **15**:297–306, 2011.