DESIGN SENSITIVITY ANALYSIS OF DYNAMIC CRACK PROPAGATION USING PERIDYNAMICS

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Summary The analytical design sensitivity analysis of dynamic crack propagation in brittle materials are discussed. Peridynamic method is employed in order to capture all main features of dynamic brittle fracture. The accuracy of derived analytical sensitivity is verified with comparison of the finite difference method.

INTRODUCTION

The problem of modeling the behavior of brittle cracks running dynamically has been under study for many decades, and yet the problem has remained largely unsolved. Here we employ peridynamics on the dynamic crack branching problem. Peridynamics is an extension of the classical continuum theory well suited for modeling of problems where discontinuities (such as cracks) initiate, propagate, and interact dynamically [1].

We derive the adjoint design sensitivity expression in peridynamics framework and apply it to the dynamic crack propagation simulation. Numerical results confirm that the analytical design sensitivity analysis (DSA) is very accurate even with evolution of dynamic cracks.

PERIDYNAMICS FORMULATION

The peridynamic formulation relies on integration of forces acting on a material point and thus it does not face any of the mathematical inconsistencies seen in the classical continuum mechanics equations. The integration takes place over a “horizon, $\mathcal{H}_{\boldsymbol{x}}$” (which, in principle, extends to infinity but, for convenience is finite) within which the material points are interacting with each other.The peridynamic equations of motion are given by:

$$\rho\ddot{\boldsymbol{z}}(\boldsymbol{x},t) = \int_{\mathcal{H}_{\boldsymbol{x}}} \boldsymbol{f}(\boldsymbol{z}(\tilde{\boldsymbol{x}},t) - \boldsymbol{u}(\boldsymbol{x},t), \tilde{\boldsymbol{x}} - \boldsymbol{x}) d\tilde{\boldsymbol{x}} + \boldsymbol{b}(\boldsymbol{x},t) \tag{1}$$

where f is the pairwise force function in the bond that connects node $\hat{\mathrm{x}}$ to x and u is the displacement vector field. ρ and $\mathrm{b(x,t)}$ are a density and a body force. The integral is defined over a region $\mathcal{H}$ called the ‘horizon’, which is the compact supported domain of f around point x.

A micro-elastic material [1] is defined as one for which the pairwise force derives from a potential ω:

$$\mathrm{f}(\eta,\xi) = \frac{\partial\omega(\eta,\xi)}{\partial\eta} \qquad \forall\eta,\xi \tag{2}$$

where $\xi = \hat{\mathrm{x}} - \mathrm{x}$ is the relative position in the reference configuration and $\eta = \hat{\mathrm{u}} - \mathrm{u}$ is the relative displacement. A linear micro-elastic potential is obtained if we take

$$\omega(\boldsymbol{\eta},\boldsymbol{\xi}) = \frac{c(\xi)s^2\|\xi\|}{2} \text{ where } s = \frac{\|\boldsymbol{\eta}+\xi\|-\|\xi\|}{\|\xi\|} \tag{3}$$

The function $\mathrm{c}(\xi)$ is called micro-modulus and has the meaning of the bond elastic stiffness. In this paper, we use the conical micro-modulus function and plane stress conditions:

$$c(\xi) = \frac{24E}{\pi\delta^3(1-\nu)}\left(1-\frac{\xi}{\delta}\right) \tag{4}$$

where E, υ, and δ are, respectively, Young's modulus, Poisson's ratio, and the radius of the horizon region.

Damage in peridynamics is implemented as the fraction between the number of broken bonds and the number of initial bonds. Each bond has a critical relative elongation s_0 at which it breaks.

ADJOINT DESIGN SENSITIVITY ANALYSIS

With the given design u, the peridynamic governing equations for $\mathbf{x}\in\Omega$ and $t\in[0,t_T]$ are given

$$\rho(\mathbf{x},u)\ddot{\mathbf{z}}(\mathbf{x},t;u) = \int_\Omega \mathbf{f}\left(\tilde{\mathbf{z}}(\mathbf{x},t;u) - \mathbf{z}(\mathbf{x},t;u), \tilde{\mathbf{x}} - \mathbf{x}, u\right) d\tilde{\mathbf{x}} + \mathbf{b}(\mathbf{x},t,u)\,, \tag{5}$$

with initial conditions

$$\mathbf{z}(\mathbf{x},0;u) = \mathbf{z}^0(\mathbf{x})\,,\quad \dot{\mathbf{z}}(\mathbf{x},0;u) = \dot{\mathbf{z}}^0(\mathbf{x}),\quad \mathbf{x}\in\Omega\,. \tag{6}$$

Consider a general integral functional (as a performance measure) in transient dynamics, in the form

$$\psi = \int_\Omega g(\mathbf{z},\dot{\mathbf{z}},u)d\Omega\Big|_{t_T} + \int_0^{t_T}\int_\Omega h(\mathbf{z},\dot{\mathbf{z}},u)d\mathbf{x}dt\,. \tag{7}$$

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Its design sensitivity [2] can be derived as

$$\psi' = \int_\Omega \left[g_{,u} \delta u \right] d\Omega \Big|_{t_T} + \int_0^{t_T} \int_\Omega \left[h_{,u} - \boldsymbol{\lambda} \cdot \left\{ \rho_{,u} \ddot{\mathbf{z}} - \mathbf{b}_{,u} - \int_\Omega \mathbf{f}_{,u} d\tilde{\mathbf{x}} \right\} \right] \delta u d\mathbf{x} dt \, , \tag{8}$$

where an adjoint system is defined as

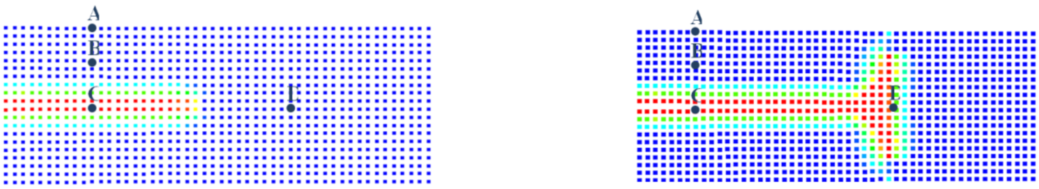
$$\ddot{\boldsymbol{\lambda}} \rho = \int_\Omega \left(\boldsymbol{\lambda} \frac{\partial \mathbf{f}}{\partial \mathbf{z}} + \tilde{\boldsymbol{\lambda}} \frac{\partial \tilde{\mathbf{f}}}{\partial \mathbf{z}} \right) d\tilde{\mathbf{x}} + h_{,\mathbf{z}} - \left(h_{,\dot{\mathbf{z}}} \right)^{\cdot} , \tag{9}$$

$$\dot{\boldsymbol{\lambda}} = -\frac{1}{\rho} \left(g_{,\mathbf{z}} + h_{,\dot{\mathbf{z}}} \right) \Big|_{t=t_T} , \ \boldsymbol{\lambda}(t_T) = \frac{1}{\rho} g_{,\dot{\mathbf{z}}} \Big|_{t=t_T} . \tag{10}$$

NUMERICAL RESULTS

Consider a pre-notched thin rectangular plate with 0.1m x 0.04m. The pre-notch is initially given as the horizontal line with 0.05m length likely a red line in Fig. 1-(a). The brittle material used is a soda-lime glass and its properties are the density of ρ = 2350 kg/m3, Young's modulus of E = 72 GPa and Poisson ratio of υ = 0.2 (υ = 1/3 in 2D bond-based peridynamics). Based on experimental results from [3], the strain energy release rate at branching is 135 J/m^2. Along upper and lower edges, tensile loadings are applied suddenly at the initial time step. Transient dynamic simulation with uniform time step 0.025 μs has been done with peridynamics and correspondingly design sensitivity analysis with respect to the displacements has been performed.

In Fig.1, crack patterns before and after dynamic crack branching are illustrated. Each particle denotes an actual peridynamic material point. Blue particles are not damaged while green-yellow-red particles are damaged sequentially. A, B, C, and D are the selected point to be verified the accuracy of their analytical design sensitivities comparing with FDM (finite difference method). Design variable is Young's modulus and its perturbation amount for FDM is 0.001% of original value. In Table 1, for both before and after branching cases, the analytical design sensitivities are agreed very well with FDM results for all selected points.



(a) Crack pattern before branching (b) Crack pattern after branching initiation

Fig. 1 Crack patterns before/after crack branching: DSA verification points (A, B, C, and D)

Table. 1 DSA verification results: comparison between FDM and DSA

Performance measure	Before branching			After branching		
	FDM	**DSA**	**Agreement**	**FDM**	**DSA**	**Agreement**
x-disp(A)	-6.18540E-06	-6.18529E-06	100.00%	7.04000E-07	7.07858E-07	99.46%
y-disp(B)	-8.97900E-06	-8.97965E-06	99.99%	-6.11310E-05	-6.11319E-05	100.00%
x-disp(C)	2.57530E-06	2.57539E-06	100.00%	3.68800E-06	3.68900E-06	99.97%
x-disp(D)	6.56560E-07	6.56454E-07	100.02%	-6.44570E-06	-6.44426E-06	100.02%
y-disp(D)	1.12780E-07	1.12794E-07	99.99%	-1.49885E-06	-1.49892E-06	100.00%

CONCLUSIONS

Peridynamics is a promising way to simulate dynamic problem with discontinuities (such as cracks). Also, an analytical design sensitivity analysis is essential for the autonomous design optimization. In this work, adjoint design sensitivity analysis of the dynamic crack propagation problem using peridynamics was performed. Even under a severe case (likely, dynamic crack branching), analytical sensitivity expression was derived and gave an excellent agreement with FDM.

References

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