GENERATION OF SET OF VANISHING ELEMENTS IN TOPOLOGY OPTIMIZATION

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Summary A new algorithm for topology optimization is proposed. It can be considered as alternative procedure to the SIMP penalization procedure. The approach is based on attention to nonlinear constraint on given volume in alternative problem formulation. A set of test problems has been solved. The obtained results are discussed.

The field of topology optimization arose recently with the advent of high-performance computers. The statement of the topology optimization problem was considered in [1], which is still fundamental in this area. One of methods is SIMP method the original idea of which was proposed by Bendsøe [2]. Since that time the methods of topology optimization developed rapidly and in recent years are widely used in design practice. Nevertheless, so far there are unsolved problems in the methods related with obtaining pure "black and white" solution. Commonly, a value of material density is introduced in such problem which specifies the percentage ratio of the used material in certain places of the design domain. Often it is difficult to interpret obtaining "grey" solution. In the paper new approach is proposed to determine which elements should be finally removed from background area.
We solve the discrete problem formulated as follows:
Min $C(x_1, x_2, \ldots, x_n)$ subject to: $\sum_{i=1}^{n} x_i = V$, where $x_i = 0$ or 1, for $i = 1, \ldots, n$, n is number of finite elements and V is given volume. The objective function C is compliance and x_i are the design variables.
In SIMP method the artificial power law is used to solve the continuous problem. In this case we minimize the function $C(x_1^p, x_2^p, \ldots, x_n^p)$. Equivalent statement of the problem can be formulated in different independent variables $y_i = x_i^p$ [3, 4]. In this case the goal function $C(y_1, y_2, \ldots, y_n)$ coincides with the goal function of initial problem. But constraint on volume transforms to nonlinear function of design variables $\sum_{i=1}^{n} y_i^{1/p} = V$.

Note that for intermediate values of y_i the value $\sum_{i=1}^{n} y_i < V$. If we take reduced value of given volume in the optimization process by SIMP method then in the intermediate step can receive the same solution as for initial problem formulation. Therefore the idea of SIMP method can be interpreted as solution without penalization but with constraint on reduced given volume. New algorithm for generation of set of vanishing elements was implemented by using this idea.
The algorithm of the topology optimization consists of the following steps:

0. Initial design variables are specified.
1. The structure, simulated by the two-dimensional finite elements, is calculated. The potential energies of the elements e_i are determined.
2. Gaussian filtering is used for e_i and for y_i.
3. New design variables are calculated with the following formulas:
   $a_i = y_i^{(k)} e_i^q$, $\sum_{i=1}^{n} \min(y^*, Ka_i) = V$, $y_i^{(k+1)} = \min(\min(y^*, 1), Ka_i)$, where k is current iteration, usually acceleration factor q is chosen between 1 to 3.
4. Sorting of design variables are performed. Maximum value of design variables among candidate elements for removing is determined. If this value less than the given value y_0 then algorithm is stopped, else steps 1–4 are repeated.
5. "Grey" elements after removing elements in step (4) are replaced on "black" elements.

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The simplest example of topology optimization is the truss illustrated in Fig. 1. The problem is to remove one rod element to obtain the structure with minimum compliance. Angle α is considered as very small. The algorithm gives correct results with removing second rod.

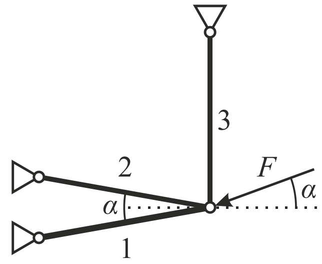


Fig. 1 – Simple truss example

The following example (Fig. 2) is taken from the paper [5]. Here two different results are presented in dependence on y^*. In the case of changeable y^* in optimization procedure the results are better but it requires more iteration N. It can be seen that these results are in good agreement with optimum results.

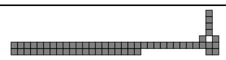
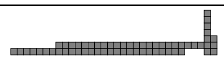
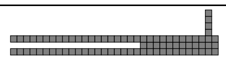
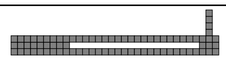
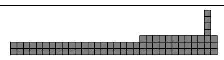
	[5]		$y^* \neq \text{const}$			$y^* = 1.5$		
V		**C**		**C**	**N**		**C**	**N**
40		558.34		559.59	67		911.76	16
60		388.66		405.17	64		409.81	15
80		255.42		256.04	231		265.60	9

Fig. 2 – Zhou-Rozvany problem

Next example is topology optimization of cantilever [6]. The proposed algorithm gives the best result in comparison with the BESO and continuation SIMP algorithms.

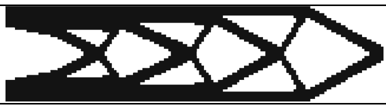
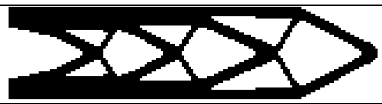
		
C=181.71, N=46, BESO	C=180.69, N=267, SIMP	C=180.68, N=24, this work

Fig. 3 – Topology optimization of cantilever

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