TOPOLOGICAL DESIGN OF ACOUSTIC MICROSTRUCTURE

Jianbin Du*a) & Ruizhen Yang*
*School of Aerospace, Tsinghua University, Beijing, 100084, China

Summary The paper deals with the problem of topology optimization of microstructure with respect to minimization of the sound power radiation from a vibrating macrostructure. The sound power is calculated in a very efficient manner using a high frequency approximation formulation. The equivalent material properties of the macrostructure are calculated using the standard homogenization method and the bi-material SIMP model is employed to achieve a zero-one design at the micro scale. Numerical examples are given to validate the approach proposed.

INTRODUCTION

In the field of structural acoustics, one of the most important issues is noise and vibration control. Unlike active control, which requires an external interference source imposed upon the system, passive control can be achieved by using sound/vibration insulation/absorbing materials, or even by direct design of sound/vibration source. For example, sound radiation from a vibrating structure may be reduced by optimization of the stiffness, mass and damping of the structure, i.e., structural and material design plays a very important part in passive control of noise and vibration. Up to now, most of the research concerning structural acoustic topology optimization has concentrated on the macro scale, i.e. the macro structural layout or material distribution [1, 2]. However, if we look into in detail some real acoustic structures or materials used in engineering, a pattern of composites with periodic microstructure can be often found. Therefore, it is natural to extend the models for structural acoustic topology optimization to the micro scale [3], or even an integration of the two scales.

The present paper is aiming at topology optimization of the periodic microstructure to minimize the total sound power radiation from the boundaries of a vibrating macrostructure. The macrostructure is excited at a single or a band of excitation frequencies by a time-harmonic external mechanical loading with prescribed amplitude and spatial distribution. The structural damping is considered to be Rayleigh damping. The microstructure composed of two different solid isotropic material phases is assumed to be identical from point to point at the macro level, which implies that the interface between the structure and the acoustic medium will not change during the design process. The upper limit of the volume fraction of the stiffer material is given in the design. Numerical examples are given to validate the model proposed. Some interesting results are revealed and discussed.

PROBLEM FORMULATIONS AND NUMERICAL RESULTS

1 Topology optimization of microstructure for minimization of the sound power radiation

Following the notation of the paper (Du and Olhoff 2007), the discretized microstructure optimization model for minimization of the total sound power flow (denoted by Π here) from the vibrating macrostructure is formulated as:

$$\begin{aligned}
&\min_{\kappa_i}\left\{\Pi = \int_S I_n dS = \int_S \frac{1}{2}\mathrm{Re}\left(p_f v_n^*\right) dS\right\}\\
&s.t.\\
&\quad \left(\mathbf{K} + i\omega_p\mathbf{C} - \omega_p^2\mathbf{M}\right)\mathbf{U} = \mathbf{P} + \mathbf{L}\mathbf{P}_f,\\
&\quad \left(\mathbf{C}_\alpha + \mathbf{H}\right)\mathbf{P}_f = \mathbf{G}\mathbf{U},\\
&\quad \sum_{i=1}^{n_e}\kappa_i V_i - V^1 \le 0,\ \left(V^1 = \gamma V_0\right),\\
&\quad 0 \le \kappa_i \le 1,\ \left(i = 1,\ldots,n_e\right).
\end{aligned} \tag{1}$$

We consider that the macrostructure is constructed by a kind of composite material with periodic microstructure, where the micro unit cell is composed of two different solid isotropic materials, and is assumed to be identical from point to point at the macro level. The micro unit cell is discretized by n_e finite elements, and the symbol κ_i denotes the relative volumetric density of the stiffer material in element i and plays the role of the design variable at the micro level. The symbol γ denotes the fraction of the given volume V^1 of the stiffer material (material 1) and is given by V^1/V_0, where V_0 is the volume of the admissible design domain of the micro unit cell. The remaining part of the total volume V_0 is occupied by a softer material (material 2). The macrostructure is constructed by periodic microstructures, thus the macro equivalent material properties can be calculated using a standard homogenization method by which the macro structural matrices in Eq. (1) can be generated (here the damping of the macrostructure is considered as Rayleigh damping). A high frequency approximation is introduced so that the sound power radiation from the boundaries of the macrostructure can be calculated in a very efficient way [1]. A SIMP based bi-material interpolation model is employed

a) Corresponding author. Email: dujb@tsinghua.edu.cn

at the micro scale to achieve a zero-one design. The sensitivity of the sound power flow of the macrostructure with respect to the micro design variable κ_i is calculated using the adjoint method and the optimization model is solved by the method of MMA.

2 Numerical examples

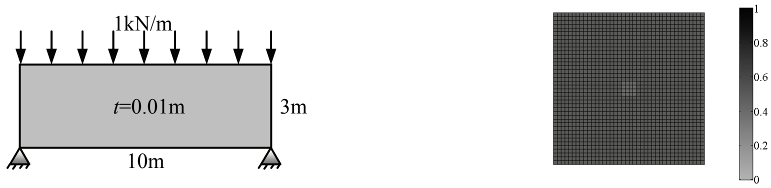

Fig. 1 Simply supported beam (a) Configuration of the macro beam; (b) Initial material distribution within the micro unit cell

The upper surface of a simply supported 2D beam is excited by a uniformly distributed time- harmonic loading, as is illustrated in Fig. . The design objective is minimization of the total sound power radiation from all the boundaries of the beam. Under the assumption of plane stress, the macrostructure and the base cell are discretized by 10×3 and 40×40 mesh respectively using the 8-node isoparametric elements. The given stiffer material has the Young's modulus $E^1 = 210\text{GPa}$, Poisson's ratio $\nu^1 = 0.3$, mass density $\eta^1 = 7800\text{kg/m}^3$ and the softer material has the properties $E^2 = E^1/10$, $\nu^2 = 0.25$ and $\eta^2 = \eta^1/10$. The mass density of the acoustic medium (i.e. air) is $\gamma_f = 1.29\text{kg}/\text{m}^3$, and the sound speed is $c = 343.4\text{m}/\text{s}$. The upper limit of the volume fraction of the stiffer material is 0.5.

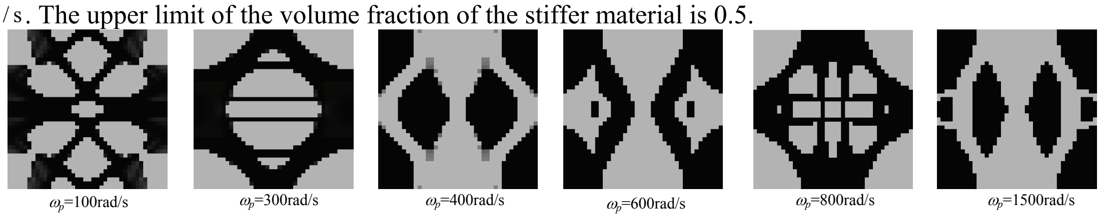

Fig. 2 Optimum topologies of the micro unit cell at six single excitation frequencies (without damping). The total sound power flow of the macro beam is minimized.

The optimum bi-material topologies of the micro unit cell with respect to six different single excitation frequencies are given in Fig. (without damping). The sound power radiation values of the initial design and the optimum design are compared in Table 1 with the unit dB transformed by $\tilde{\Pi}(\text{dB}) = 10 \cdot \lg(\Pi/\Pi_0)$, where $\Pi_0 = 1\times10^{-12}\text{W}$ is a reference value.

Table 1 Sound power radiation values of the initial design and optimal design

Excitation Frequency(rad/s)	Initial Design(dB)	Optimal Design(dB)
100	88.22	82.58
300	112.69	98.35
400	105.61	98.21
600	95.71	90.56
800	95.30	88.44
1500	89.01	83.08

Multiple frequency designs are also performed in the frequency intervals ω_p=200~1000rad/s (9 uniformly distributed sample points) and ω_p=3000~5000rad/s (11 uniformly distributed sample points), respectively. The optimum topologies of the micro unit cell are given in Fig. 3(a) and 3(b). The corresponding frequency response curves of the optimum designs and the initial design are given in Fig. 3(c).

(a) (b) (c)

Fig. 3 Optimum topologies of the micro unit cell by multiple frequency optimization (the Rayleigh damping coefficients of the macro structure α=1×10^{-2}, β=1×10^{-4}) (a) ω_p=200~1000rad/s (b) ω_p=3000~5000rad/s (c) frequency response curve

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References

[1] Du J., Olhoff N.: Minimization of sound radiation from vibrating bi-material structures using topology optimization. *Struct Multidisc Optim*. **33(4-5)**: 305-321, 2007.

[2] Du J., Olhoff N.: Topological design of vibrating structures with respect to optimum sound pressure characteristics in a surrounding acoustic medium. *Struct Multidisc Optim*. **42(1)**: 43-54, 2010.

[3] Sigmund O.: Materials with prescribed constitutive parameters: An inverse homogenization problem. *International Journal of Solids and Structures* **31(17)**: 2313-2329, 1994.