Multi-scale Design and Optimization Considering Load Uncertainties

Xu Guo[*], Guomin Sun
[*]*Department of Engineering Mechanics, Dalian University of Technology,*
State Key Laboratory of Structural Analysis for Industrial Equipment
Dalian, 116023, China

Summary Uncertainty is ubiquitous in practical engineering design applications. Recent years have witnessed a growing research interest in the study of structural topology optimization problems considering uncertainties. Most of these works, however, are focused on the optimization of macro-scale structures. In the present work, concurrent optimization of material and structure under load uncertainties are studied in a multi-scale optimization framework. Mathematical formulation that can considered the effect of uncertainties in a confidence way and the corresponding solution procedure for the considered problem are proposed. Numerical examples examined demonstrate the effectiveness of the proposed solution approach. It is found that the material distribution in the microstructures tends to be isotropic and the Kagome structure is superior compared with other forms of microstructure when load uncertainties are considered.

INTRODUCTION

Uncertainty is ubiquitous in practical engineering design applications. Recent years have witnessed a growing research interest in the study of structural topology optimization problems considering uncertainties. Most of these works, however, are focused on the optimization of macro-scale structures. It is well known that current optimization of the macro-scale shape of structure and the material distribution in its microstructure can be helpful for utilizing the material with much higher efficiency, therefore it is of great importance to discuss how to carry out structural topology optimization in a multi-scale framework with uncertainties being taken in to consideration, which is just the main topic of the present study.

PROBLEM FORMULATION

The considered optimization problem is to find the material distribution in the micro- and macro scales currently with the aim of maximizing the worst case stiffness (measured by the mean compliance of the structure under the most unfavoriate loading condition) under available material volume constraint. The uncertainty of the external load is described by the ellipsoid model. In order to perform topology optimization in a multi-scale framework, as in [1] and [2], two types of the design variables $\rho(\boldsymbol{x})$ and $\mu(\boldsymbol{y})$, are introduced to depict the material distributions in macro- and micro-scale, respectively, see Fig.1 for reference. Note that in order to make the solution process tractable, in the present study, it is assumed the microstructure at every material point of the structure takes the same form.

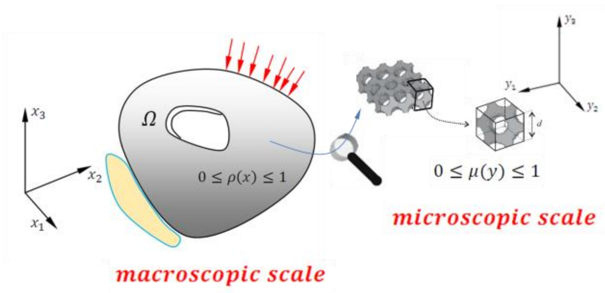

Fig.1 Material distribution in micro and macro scales.

Based on the above assumptions, the corresponding worst case current optimization problem can be formulated in the following Bi-level form:

$$\wp_{in}$$

$$\begin{gathered}
\text{find } \rho(\boldsymbol{x}) \in \mathrm{L}^{\infty}(\mathrm{D}), \mu(\boldsymbol{y}) \in \mathrm{L}^{\infty}(\mathrm{Y}) \\
\min C_{\rho,\mu} = global\ optimum\ of\ \wp_{out} \\
\text{s.t.} \\
\frac{1}{|\mathrm{D}|}\int_{\mathrm{D}} \rho_0 \rho(\boldsymbol{x}) \mathrm{dV} \le \varsigma, \quad 0 < \rho_{\min} \le \rho(\boldsymbol{x}) \le 1, \\
\frac{1}{|\mathrm{Y}|}\int_{\mathrm{Y}} \mu(\boldsymbol{y}) \mathrm{dY} = \rho_0, \quad 0 < \mu_{\min} \le \mu(\boldsymbol{y}) \le 1
\end{gathered} \tag{1}$$

and

$$\wp_{out}$$

$$\text{find } \bar{\boldsymbol{t}} \in \mathrm{U}_{\bar{t}}, \boldsymbol{u}(\boldsymbol{x}) \in [\mathrm{H}^1(\mathrm{D})]^3$$

$$
\begin{aligned}
&\max\ \ l(\bar{\boldsymbol{t}},\boldsymbol{u}) = \int_{\mathrm{S_t}} \bar{\boldsymbol{t}}\cdot\boldsymbol{u}\mathrm{dS} \\
&\mathbf{s.t.} \\
&\int_{\mathrm{D}} [\rho(\boldsymbol{x})]^p \mathbf{E}^{\mathrm{H}}:\nabla\boldsymbol{u}:\nabla\boldsymbol{v}\mathrm{dV} = \int_{\mathrm{S_t}} \bar{\boldsymbol{t}}\cdot\boldsymbol{v}\mathrm{dS},\quad \forall\boldsymbol{v}\in\mathrm{U_{ad}}, \\
&\boldsymbol{u}=\bar{\boldsymbol{u}},\ \ \text{on } \mathrm{S_u}
\end{aligned}
\tag{2}
$$

with

$$
\mathbf{E}^{\mathrm{H}}:\nabla\boldsymbol{u}:\nabla\boldsymbol{u} = \min_{\boldsymbol{\chi}-\mathrm{Y_{per}}} \frac{\mathbf{1}}{|\mathrm{Y}|}\int_{\mathrm{Y}} [\mu(\boldsymbol{y})]^p \mathbf{E}^0:\nabla(\boldsymbol{u}-\boldsymbol{\chi}^*):\nabla(\boldsymbol{u}-\boldsymbol{\chi}^*)\mathrm{dY}. \tag{3}
$$

In Eq. (1), ρ_0 and ς denote the porosity and upper bound of the volume fraction of the available material, respectively. The symbol $\boldsymbol{\chi}^*$ in Eq. (2) denotes the character displacement field associated with the microstructure whose material distribution is depicted by $\mu(\boldsymbol{y})$. $\mathbf{E}^0$ is the elasticity tensor of the solid material and p is a penalty parameter. D and Y denote the design domain on the macro- and micro-scale, respectively. $\mathrm{S_u}$ and $\mathrm{S_t}$ are the displacement prescribed and traction prescribed boundary of macro-scale structure, respectively. $\rho_{\min}$ and $\mu_{\min}$ denote the lower bound of ρ and μ, respectively.

In the above Bi-level problem formation, the upper level program in Eq. (1) is used to find the optimal values of $\rho(\boldsymbol{x})$ and $\mu(\boldsymbol{y})$ while the lower level program in Eq. (2) is used to find the worst loading case under given values of $\rho(\boldsymbol{x})$ and $\mu(\boldsymbol{y})$. It is worth noting that the objective function in Eq. (1) requires the global optimum of the lower level program while the lower level program itself is non-convex in nature. Under this circumstance, it is highly possible that the solution of the lower level problem will get stuck at a local optimum and therefore underestimates the worst case structural response. In order to overcome this difficulty, A SDP-relaxation of the lower level program is carried out. By doing so, the confidence of the optimal solution can be fully guaranteed. For the limitation of space, the detailed description of the solution procedure is omitted.

NUMERICAL EXAMPLE

With use of the proposed problem formulation and the solution procedure, the well-known L-shape beam example shown in Fig. 2is examined under load uncertainty. ρ_0 and ς in this example are 0.3 and 0.4, respectively. The Young's modulus and Poisson's ratio of the isotropic solid material are 1.0 and 0.3, respectively. The uncertainty of the external load is described by $(\boldsymbol{f}-\boldsymbol{f_0})^{\top}\boldsymbol{B}(\boldsymbol{f}-\boldsymbol{f_0})-1\le 0$ with $\boldsymbol{B}=\begin{bmatrix}16 & 0\\ 0 & 16\end{bmatrix}$ and $\boldsymbol{f_0}=(0,0.5)^{\top}$, respectively. Fig. 3 and Fig. 4 show the optimal distribution on different length scales under uncertainty load and deterministic load $\boldsymbol{f_0}$, respectively. Both $\rho_{\min}$ and $\mu_{\min}$ are all set to 1.0e-03 in the numerical implementation.

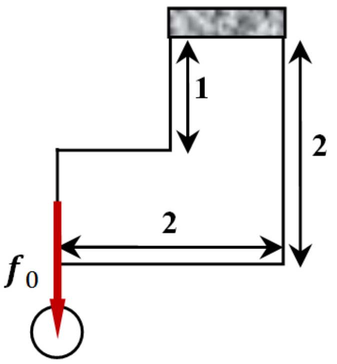

Fig. 2 L-shape beam example

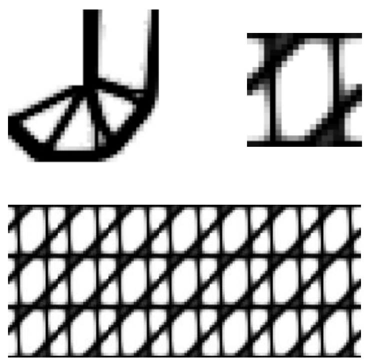

Fig. 3 Optimal material distribution under nominal load $\boldsymbol{f}_0$

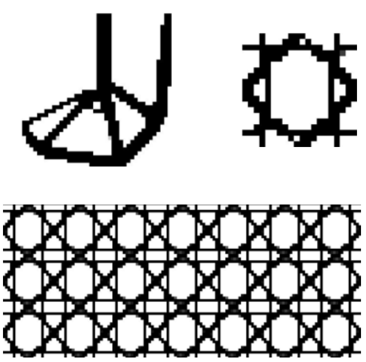

Fig. 4 Optimal material distribution under uncertain load

CONCLUSIONS

In the present work, concurrent optimization of material and structure under load uncertainties are studied in a multi-scale optimization framework. Mathematical formulation that can considered the effect of uncertainties in a confidence way and the corresponding solution procedure for the considered problem are proposed. Numerical examples examined demonstrate the effectiveness of the proposed solution approach. It is found that the material distribution in the microstructures tends to be isotropic and the Kagome structure is superior compared with other forms of microstructure when load uncertainties are considered.

References

[1] Rodrigues H., Guedes J.M., Bendsøe M.P.: Hierarchical optimization of material and structure. *Struct. Multidisc. Optim* **24:**1-10, 2002.

[2] Liu L., Yan J., Cheng G.D.: Optimum structure with homogeneous optimum truss-like material. *Computers and Structures* **86**:1417-1425, 2008.