DESIGN FOR MAXIMUM BAND-GAPS IN BEAM STRUCTURES

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Summary This paper aims to extend earlier optimum design results for transversely vibrating Bernoulli-Euler beams by determining new optimum band-gap beam structures for (*i*) different combinations of classical boundary conditions, (*ii*) much larger values of the orders *n* and *n*-1 of adjacent upper and lower eigenfrequencies of maximized band-gaps, and (*iii*) different values of a minimum cross-sectional area constraint. The periodicity of the optimum beams and the attenuation of their band-gaps are also discussed.

INTRODUCTION

A band-gap structure usually consists of a *periodic* distribution of elastic materials or segments, where the propagation of waves is impeded or significantly suppressed for a range of external excitation frequencies. Maximization of the band-gap is therefore an obvious objective for optimum design. This problem is sometimes formulated by optimizing a parameterized design model which assumes periodicity in the design. However, it is shown in the present paper that such an *a priori* assumption is not necessary since, except for regions adjacent to the structural boundaries, the maximization of the band-gap alone, generally leads to significant design periodicity.
The band-gap phenomenon may occur for elastic, acoustic or electromagnetic waves, cf. Refs. [1-4]. Due to a wealth of applications in vibration protection, noise isolation, waveguiding, etc., the study and development of band-gap mass-spring, beam, grillage, disk and plate structures, in most cases by topology optimization, have attracted increasing attention in recent years, see papers [4-9] and references cited therein. Refs. [10,11] seem to be the first papers dealing with problems of optimizing vibrating structures for maximum band-gaps – *albeit* the term difference (or separation) between adjacent eigenfrequencies was used in these papers.
In the present paper, instead of maximizing band-gaps between frequencies of propagating waves or forced vibrations excited by external time-harmonic loads, the closely related problem of maximizing the gap between two adjacent eigenfrequencies ω_n and ω_{n-1} of any given consecutive orders *n* (*n*>1) and *n*-1, is considered. This is justified by the fact that external time-harmonic dynamic loads cannot excite resonance with high vibration levels of standing waves, if the eigenfrequencies of the structure are moved outside the range of the external excitation frequencies by the optimization. Moreover, the present study has shown that, if an infinite beam structure is constructed by repeated translation of an inner beam segment obtained by the aforementioned frequency gap optimization, then a band-gap of traveling waves in this infinite beam is found around the maximized frequency separation.

APPROACH

The beams have given length *L* and volume *V*, are made of a linearly elastic material with Young's modulus *E* and mass density γ and no damping, and they have variable, but geometrically similar (e.g., circular) cross-sections with the relationship $I=cA^2$ between the area moment of inertia *I* and the cross-sectional area *A*, where the constant *c* is given by the cross-sectional geometry. The optimization problem is considered in non-dimensional form, where α denotes the cross-sectional area function (the design variable), and the dimensionless *n*-th circular eigenfrequency ω_n for free, transverse vibrations is defined by $\omega_n^2 = \bar{\omega}_n^2 \gamma L^5 / \left(cEV\right)$, where $\bar{\omega}_n$ is the dimensional *n*-th circular eigenfrequency of the beam. A bound formulation [12] with two variable bound parameters is set up for the problem of maximizing the gap between the two eigenfrequencies ω_n and ω_{n-1}, and the problem is solved by finite element- and gradient-based optimization using analytical sensitivity analysis. For this, an incremental optimization scheme is developed, that is applicable for problems with any mix of multiple and simple eigenfrequencies [8].

RESULTS

Figs. 1(a-d) present examples of cantilever beams with maximized higher order eigenfrequency gaps $\Delta\omega_3 = \left(\omega_3 - \omega_2\right)$, $\Delta\omega_4 = \left(\omega_4 - \omega_3\right)$, $\Delta\omega_{19} = \left(\omega_{19} - \omega_{18}\right)$ and $\Delta\omega_{20} = \left(\omega_{20} - \omega_{19}\right)$ by way of depicting the linear dimensions $\pm\sqrt{\alpha}$ perpendicular to the beam axes to a suitable scale. A lower limit $\alpha_{\min}$ =0.05 is prescribed for the cross-sectional area.

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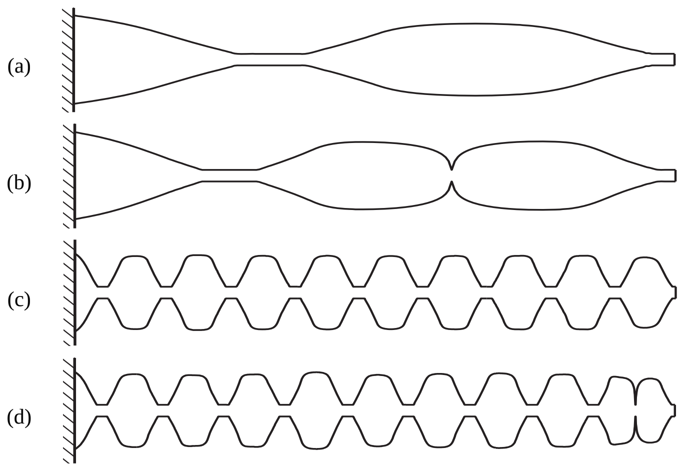

Fig.1 Optimized cantilever beams: (a) maximized frequency gap $\Delta\omega_3$ = 129.72, $\Delta\omega_3 / \Delta\omega_3^u$ = 3.27; (b) maximized frequency gap $\Delta\omega_4$ = 195.15, $\Delta\omega_4 / \Delta\omega_4^u$ = 3.30; (c) maximized frequency gap $\Delta\omega_{19}$ = 5141.39, $\Delta\omega_{19} / \Delta\omega_{19}^u$ = 14.47; (d) maximized frequency gap $\Delta\omega_{20}$ = 5542.40, $\Delta\omega_{20} / \Delta\omega_{20}^u$ =14.78.

As a convenient reference for the examples, we denote by $\Delta\omega_n^u$ the gap between the *n*-th and (*n-1*)-st eigenfrequencies of a uniform comparison beam with the same boundary conditions, length, volume, material, and cross-section parameter *c* as the optimized beams. The absolute values of the optimized frequency gaps and their relative values with respect to those of the uniform comparison beams are given in the caption of Fig. 1. Several optimization results for other sets of classical boundary conditions, different values of the orders of the frequency gaps, and different values of the minimum constraint $\alpha_{\min}$ have been determined.

Discussion

The designs optimized for maximum $\Delta\omega_3$ and $\Delta\omega_4$ in Figs. 1(a) and 1(b), respectively, are almost indistinguishable from optimum designs obtained by a different approach in [10] for the corresponding problems with $\alpha_{\min}$ =0; compare Figs. 1(a) and 1(b) with Figs. 5 and 6 in [10].

The examples in Figs. 1(a-d) reflect some general characteristics found in the present study: (*i*) The optimized bandgaps are large relative to those of the uniform comparison beams, and they increase quite substantially with the order *n* of the bandgap. (*ii*) Generally, already starting from a moderate value of *n*, like *n*=5 or 6, the optimized beam designs exhibit periodicity, and the periodicity increases significantly with increasing values of *n*. (*iii*) Deviation from design periodicity has only been found in regions adjacent to the structural boundaries, and the type depends on the boundary conditions.

Remark on wave propagation and forced vibration

To study the band-gap for travelling waves, a repeated inner segment of the optimized beams was analyzed using Floquet theory and the waveguide finite element (WFE) method. The frequency response was then computed for the optimized beams when these were subjected to an external time-harmonic loading with different excitation frequencies, and it was found that the attenuation levels were very significant in the prescribed frequency band-gaps.

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