ISSUES RELATED TO TOPOLOGY OPTIMIZATION OF SNAP-THROUGH PROBLEMS

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Summary This work focuses on issues related to topology optimization of static geometrically nonlinear structures experiencing snap-through behaviour. Different compliance and buckling criterion functions are studied and applied to topology optimization of a point loaded curved beam problem with the aim of maximizing the snap-through buckling load.

INTRODUCTION

Since the pioneering work by [1], numerous papers dealing with topology optimization have been published and stiffest structural design by topology optimization has today become an automated task implemented in commercial software. Current research in topology optimization are therefore directed towards extension of the method to more complex applications with more difficult performance criteria. One of the most important objectives/constraints in structural optimization is stability (buckling), and this also holds for topology optimal designs that often have the nature of thin frame-like structures that inherently are prone to buckling.

This paper deals with the issues related to topology optimization of static geometrically nonlinear snap-through problems when the aim is to increase the structural buckling load for a given mass. Specifically, we study different objective functions in terms of compliance and buckling measures for improvement of the snap-through buckling load. A 2D curved beam problem that exhibits snap-through behavior is used as an example of a generic buckling problem.

A well-known issue in topology optimization when applying buckling criteria is artificial buckling modes in low density regions. The typical cure applied for linear buckling is to penalize the stiffness terms within the buckling problem differently, see [2, 3]. However, this approach does not have a natural extension to nonlinear problems, as it results in inconsistencies within the static problem and the design sensitivity analysis, and we propose an alternative approach.

This study also highlights some possible negative implications of using symmetry to reduce the model size and it is demonstrated how an initial symmetric buckling response characterized by limit point buckling may change to an asymmetric bifurcation buckling response during the optimization process. This problem may partly be avoided by not exploiting symmetry, however special requirements are needed of the analysis method and optimization formulation.

METHODS

The finite element method is used for determining the structural response of the structure. A range of different objective functions are applied in an attempt to increase the buckling resistance during topology optimization. These include linear compliance, nonlinear end compliance, linear buckling, and nonlinear buckling.

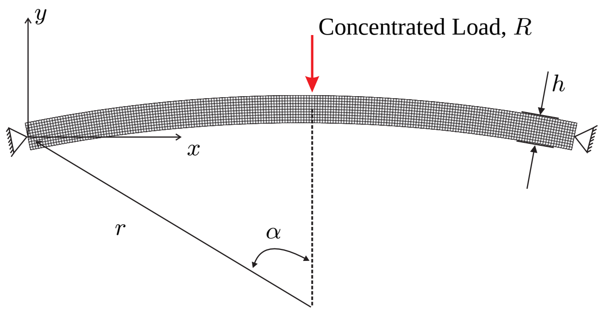
Static linear and geometrically nonlinear analysis are applied for the different criteria and a range of different stop criteria are used for the nonlinear analysis in order to investigate the importance of geometrically nonlinear effects.

The linear buckling load is predicted by linearized eigenbuckling analysis by assuming linear behaviour and neglecting prebuckling displacements. The nonlinear buckling criterion is based upon geometrically nonlinear static analysis and an extended eigenbuckling problem is formulated and solved in the vicinity of the buckling load. In order to avoid problems with mode switching during the optimization process a number of the lowest buckling loads is considered via the bound formulation. See [4] for further details on the design sensitivity analysis and applied optimization formulation.

The mathematical programming problems are solved by the gradient based optimizer MMA by [5].

EXAMPLE AND RESULTS

A generic plane stress buckling problem of a curved beam, see figure 1, is applied in this study.



$r = 5.05mm$	$h = 0.1m$
$t = 10mm$	$\alpha = 0.1993rad$
$L = 2m$	$R = 10kN$
$E = 210GPa$	$\nu = 0.3$

Figure 1. Geometry, loads, boundary conditions, and material properties for the 2D curved beam problem.

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Standard 2D 8-noded quadratic isoparametric displacement based finite elements are used to model the beam. The mass constraint for the topology optimization is set to 50% of the initial design. The structural response of the initial grey topology design with all element densities set to $\rho_e = 0.5$ is shown in figure 2. The stability of the initial design is governed by a snap-through limit point instability with a symmetric buckling mode behaviour. Asymmetric buckling response in the form of bifurcation-type instabilities are not present. Therefore, symmetry is enforced to reduce the model size for the optimization studies.

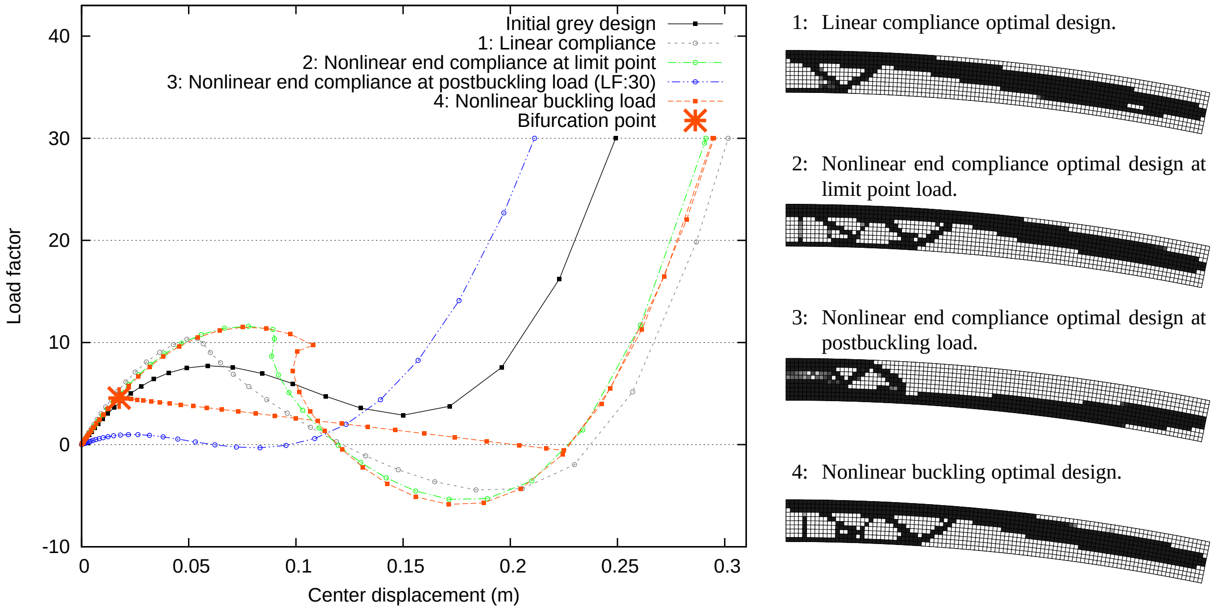

Figure 2. Load-deflection curves for optimum designs obtained by geometrically nonlinear analysis.

Optimum topologies and their final equilibrium curves from the topology optimization studies on a half model are shown in figure 2. The optimum topologies for optimizations 1-3, which all considers compliance type objective functions, are very different and so are their structural performance. Optimization 2, which minimizes nonlinear end compliance at the limit load, has a higher buckling load than optimization 1 which minimizes linear compliance. Optimization 3 minimizes nonlinear end compliance in the postbuckling regime at a load factor of $LF = 30$. This topology design is completely different from those from optimization 1 and 2 and has a much lower snap-through buckling load but a much higher postbuckling stiffness. This clearly shows the importance of geometrically nonlinear effects and that special care should be devoted to choosing a proper load level for the stiffness optimization. Finally, the limit point buckling load has been optimized directly by the nonlinear buckling criterion in optimization 4. The topological design and structural performance is very similar to optimization 2, thus minimum nonlinear end compliance at the limit load seems to be a good criterion to improve the buckling performance.

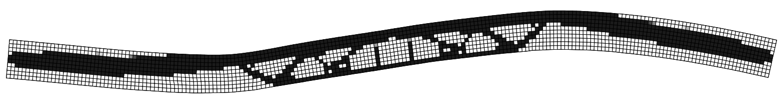

Figure 3. Asymmetric bifurcation buckling mode of optimal design from optimization 4.

To validate the design, a full model has been generated by mirroring the topological design from optimization 4 and a complete geometrically nonlinear assessment is carried out. Surprisingly, it turns out that a new stability point of the bifurcation type has been introduced during the optimization such that the buckling load is less than half of the limit point buckling load. The loading point and buckling mode for the bifurcation point are shown in figure 2 and 3, respectively.

CONCLUSIONS

It has been demonstrated that topology optimization of geometrically nonlinear structures for increased buckling performance is a challenging task. We have shown that special requirements are needed of the analysis method and optimization formulation in order to reliable deal with this problem.

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