SINGULAR PHENOMENON OF INTEGRATED SIZE AND TOPOLOGY OPTIMIZATION OF FRAME STRUCTURES WITH EXACT FREQUENCY CONSTRAINTS

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Summary The integrated size and topology design of frame structures for minimum weight subjected to exact frequency constraints is studied. The feasible domain is found to be disjointed and 'singular optimum' may exist, which means the optimum design may be on the tip of the separated sub-domain of the feasible domain. A new kind of material interpolation 'PoLynomial Material interPolation(PLMP)' is proposed on the beam stiffness and mass which increases the possibilities to find the "singular optimum" with the gradient based algorithm.

INTRODUCTION

For the design of frame structures, it is often required that the corresponding natural frequencies should escape from the frequencies or frequency bandwidths of the exciting forces in order to avoid severe structural resonance. This can be achieved by adjusting the eigen-frequency spectrum of the structure with simulation-based structural optimization techniques. Within this context, a natural thought is to formulate the problem as a weight minimization problem with frequency constraints using the cross-sectional areas of the beams in frame structures as design variables.
Compared to size optimization, the structural topology optimization[1-2]can determine the optimal structural connections of the nodes, elements, and the internal boundaries and give out the best loads transmission path for a given design domain, loading boundary and functional requirements. In recent years, topology optimization on structural dynamic properties has achieved great developments[3-5]. But when we use the ground structure method to study the light design of frame structure with frequency constraints, local vibration frequency of one or one group of beams may become the fundamental frequency and violate the specified frequency constraints with the decreasing of the beam's cross-sectional characteristic parameters. This will lead the result that these "slim" bars with low structural efficiency can't be deleted from the original structure and the structural topology can't be optimized. This is to say, we can't carry out the topology optimization of the frame structure with frequency constraint with size optimization smoothly. Meantime some studies[4-5] found the local vibration had obvious influence on the optimal design, but lack of systematic exposition for the nature of this phenomenon.
In this paper, we adopt dynamic stiffness matrix method[6] together with w-w algorithm and Newton algorithm to obtain the exact natural frequency of the frame structure. The shape of the feasible domain of a three-beam frame with fundamental frequency constraint is described. 'Singular optimum' phenomenon is observed, and then the relation between the local vibration mode and the singular optimum is researched. A new kind of interpolation 'PoLynomial Material interpolation (PLMP)' is proposed to improve the connectivity of the feasible domain effectively, and therefore increases the possibilities of finding the singular optimum with gradient based algorithm.

OPTIMIZATION FORMULATION

Assuming analogical cross-sections of frame beams and choosing the beam cross-sectional area as the design variable, minimum weight design of frame with frequency constraints can be formulated as follows.

$$\begin{cases} to\ \ find\ \mathbf{A}=\{A_i\ ,i=1,...,m\} \\ Min\quad W=\sum_{i=1}^{m}\rho_i A_i L_i \\ \quad s.t. \qquad \mathbf{K}(\boldsymbol{\omega},\mathbf{A})\mathbf{u}=0 \\ \qquad\qquad \min_{j=1,\dots,J}\{\omega_j(\mathbf{A})\}-\underline{\omega}\ge 0 \\ \qquad\qquad 0\le A_i\le \overline{A_i}\ \ ,i=1,2\cdots m \end{cases} \tag{1}$$

where **A** is the vector of design variables composed of m beams' cross-sectional areas. **K** is the dynamic stiffness matrix whose detailed formula can be found in reference[6]. **u** is the eigenvector. ω_j is the j-th frequency of the frame with the lower frequency bounds $\underline{\omega}$. $\rho_i, A_i, L_i, i=1,2\ldots m$ are the material mass density, cross-sectional area and length respectively of i-th beam. Here, we allow the design variables **A** with lower bound zero to realize the changes of structural topology.

THE FEASIBLE DOMAIN OF 3-BEAM EXAMPLE

Here we discuss the feasible domain of optimization problem of 3-beam frame with fundamental frequency constraint of 50Hz shown in Figure 1. Assuming the beam cross-section are circle, and the two beams numbered A_2 have the [a]

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same cross-section areas. The shape of the feasible domain is described in Figure 1. In Figure 1, the green area and the two green rays on the coordinate axis are the feasible domains of the optimization problem. We can observe from Figure 1 that the feasible domain includes two degraded feasible subdomains (two green rays on the axis) disconnected to the main feasible domain. By drawing a set of contours of the weight function in Figure 1, we can see that the cut-off point of the contours with the main feasible domain corresponds to (A_1,A_2)=(8.76, 4.75) cm^2 with a weight of 17.31Kg is only a local optimal solution. The global optimal solution is located at the intersection of contours with the vertical axis with a weight of 16.64 Kg where (A_1,A_2)=(0,7.54) cm^2. In this case, A_1 = 0, the original three-beam frame degenerates into a two-beam frame. This observation confirms the existence of singular optimum for the integrated size-topology optimization of frames with frequency constraints. Such singular phenomenon origins from the disjointed feasible sub-domain, which is due to the switch of vibration modes at the critical point of structural topology change.

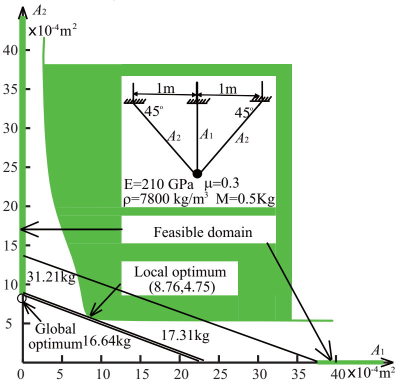

Figure 1 Feasible domain and contour with 50Hz constraint

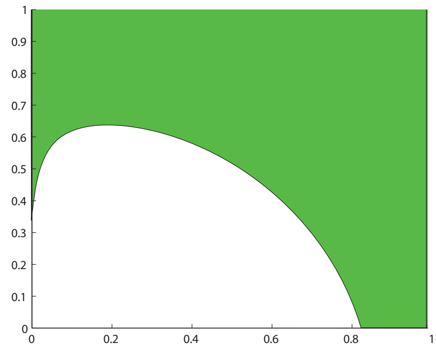

Figure 2 Feasible domain with $a = 0.999$ in Eq.2

NEW MATERIAL INTERPOLATION FOR SIZE-TOPOLOGY OPTIMIZATION

To cope with the singular difficulty and improve the possibility of finding the global optimum, we propose a new material interpolation--PLMP (PoLynomial Material interPolation). The cross-sectional area and moment of inertia of the i-th beam are artificially defined as,

$$A_i = f(x_i)A_0 = x_i A_0$$
$$I_i = g(x_i)A_0^2 = \alpha A_0^2 (a x_i^2 + (1-a)x_i) \quad , a \in [0,1] \tag{2}$$

where x_i is the artificially interpolation variable, $f(x_i), g(x_i)$ are the interpolation functions of the cross-sectional area and moment of inertia. α is constant related with the shape of cross section, for example $\alpha = 1/4\pi$ when the cross-section is circle. Note that the interpolation recovers the physical relation between cross sectional area and moment of inertia in case of $a = 1$.

Adopting the new interpolation formula in equation 2, we can draw the different feasible domains of the three-beam frame example with 50Hz bound of frequency constraints when parameter a equals different values. The feasible domain with a=0.999 is shown in Figure 2. We can see from Figure 2 the use of PLMP interpolation significantly changes the shape of the feasible domain, and even the connectivity of the feasible domain. Meanwhile, observing Figure 2, we can see that the feasible domain from PLMP model can give a good approximation of the singular optimum on x_1 axis and also ensure good connectivity when the parameter a tends to 1. This will greatly increase the probability to find the global optimum with gradient based algorithms. The effectiveness of the new material interpolation and the optimization model are demonstrated through two numerical examples of 10-beam frame and 35-beam frame which will be presented on the conference if it is possible.

CONCLUSIONS

We use the dynamic stiffness method to calculate the frequencies of frame structure exactly in this paper. We find the global optimum may lie on the endpoints of a separate degraded sub feasible domain. That is a new kind of 'singular' optimum. In order to overcome this difficulty, we introduce artificial interpolation variable as the design variable, and suggest a new type of material interpolation (PLMP). It is found the PLMP interpolation can increase the possibilities of finding the "singular" optimal topology effectively, and realize the integrated structural size and topology optimization.

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