THICKNESS OPTIMIZATION OF FIBER REINFORCED LAMINATED COMPOSITES USING THE DISCRETE MATERIAL OPTIMIZATION METHOD

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Summary This work concerns a novel large-scale multi-material topology optimization method for simultaneous determination of the optimum variable integer thickness and fiber orientation throughout laminate structures with fixed outer geometries while adhering to certain manufacturing constraints. The conceptual combinatorial/integer problem is relaxed to a continuous problem and solved on basis of the so-called Discrete Material Optimization method, explicitly including the manufacturing constraints as linear constraints.

INTRODUCTION

Fiber reinforced laminated composites are applied in a variety of high performance structures because the stiffness to weight ratio surpasses e.g. steel and aluminum alloys. The basis of this work is laminated structures where the outer geometry for some reason is fixed which could be the case for e.g. wind turbine blades where aerodynamics may govern the shape of the outer surface. Weight reduction while maintaining sufficient stiffness of such structures implies that the thickness of the laminate must vary inwards in an optimal manner. The individual plies/layers that together constitute the laminate are typically chosen among a finite set of materials with finite sets of fiber angles, e.g. Carbon/Glass Fiber Reinforced Polymer (CFRP/GFRP) with $\{0°, +45°, -45°, 90°\}$ fiber orientation. The task for the designer is simultaneously to determine the optimum variable integer thickness and fiber orientation throughout the laminate structure. The problem is therefore conceptually a combinatorial multi-material topology optimization problem. This short paper presents a novel method that treats this problem. The optimized design is typically subject to certain manufacturing constraints (MC): To accommodate application of predefined fiber mats, large regions/patches within the layers of the total geometry/laminate must be identical (MC1); to prevent failure such as delamination and matrix cracking problems, changes in thickness cannot be too abrupt (MC2) and the number of identical contiguous plies should not be too high (MC3). Explicit inclusion of these manufacturing constraints in the topology optimization problem while preventing intermediate void in the monolithic laminate yields an optimized starting point for detailed post processing before true realization may take place. Stress constraints and failure criteria are not considered explicitly in this work. Preventing intermediate void and fulfilling MC1-MC3 will, however, implicitly reduce the risk of failure.
Figure 1 illustrates a simplified 1D example, discretized as six Equivalent Single Layer (ESL) shell elements (one ESL shell element per column), each containing three layers (rows).

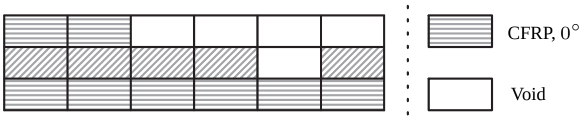

Figure 1: Example of a thickness optimized laminate. The bottom layer is enforced material; free from void.

The example in Figure 1 could represent the solution to an optimization problem subject to the following requirements: Bottom layer is enforced material (i.e. free from void); choose between void and $\{0°, +45°, -45°, 90°\}$ CFRP; maximum allowable mass fraction is 13/18; intermediate void is not allowed; besides void there must be fiber angle continuity in an entire row/layer (MC1); total allowable thickness change is $\pm$ 1 ply thickness between neighbouring ESL shell elements (MC2); only a single contiguous identical layer/ply is allowed (MC3).
Recent work on simultaneous material distribution and orientation has been summarized in [2]. Although heuristic and genetic algorithms are frequently applied and in principle well suited for combinatorial/integer problems, they do, however, rely on extensive amounts of finite element analyses. Despite the ever-increasing parallelizable computational power, it is unacceptably expensive to apply such methods for large-scale optimization problems. In this short paper, the combinatorial/integer thickness optimization problem is relaxed thus allowing application of gradient based state-of-the-art optimizers as e.g. SNOPT [3] that is used in this work. The problem is solved on basis of the so-called Discrete Material Optimization method, see [4]. Prevention of intermediate void and MC1-MC3 are explicitly included in the optimization problem as a large number of sparse linear constraints.

PROBLEM FORMULATION

The method to be presented takes basis in multi-phase compliance minimization through the density approach, see [1], in which the constitutive properties of all layers in all ESL shell elements in the discretized domain are functions of the densities (design variables).

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$$\boldsymbol{E}_{ij} = \boldsymbol{E}_0 + \frac{\rho_{ij}}{1+p(1-\rho_{ij})} \sum_{k=1}^{n^c} \frac{x_{ijk}}{1+q(1-x_{ijk})} (\boldsymbol{E}_k - \boldsymbol{E}_0) \quad , \quad \forall (i,j) \tag{1}$$

In (1), index i concerns a specific ESL shell element, index j concerns a specific layer, and index k concerns a specific material candidate. $\boldsymbol{E}_k$ is the constitutive matrix for material candidate k, $\rho_{ij} \in [0,1]$ are topology variables (densities), $x_{ijk} \in [0,1]$ are candidate material variables, p and q are penalization powers for the variables, and n^c is the number of specific candidate materials. $\boldsymbol{E}_0$ is massless and significantly more compliant than all $\boldsymbol{E}_k$. The parameterization in (1) utilizes the material interpolation scheme RAMP (Rational Approximation of Material Properties), see [5]. A result to the topology optimization problem is satisfactory when: Compliance is reduced; all variables are binary; a mass constraint and the manufacturing constraints (MC1-MC3) are satisfied; intermediate void is prevented.

EXAMPLE AND RESULTS

Figure 2 illustrates the setup, a 2D cantilever plate. Because of symmetry, only half of the plate is considered (grey). The grey plate is discretized as 39×26 square ESL shell elements in seven layers. A layer has thickness $t = 1mm$.

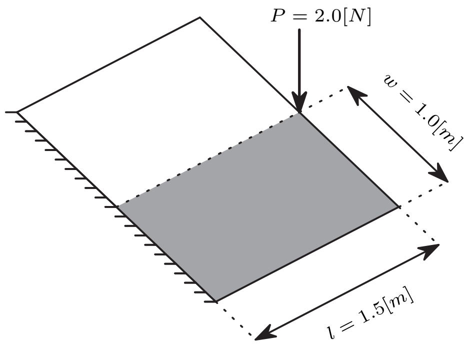

Figure 2: Setup, a 2D cantilever plate.

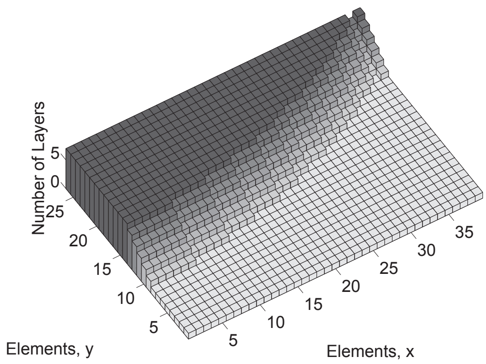

Figure 3: Thickness optimized laminate.

The optimization problem is linearized and solved sequentially, using SNOPT with an adaptive move limit update scheme. The problem is solved in two consecutive steps. Step 1 converges without penalization. Step 2 continues with constant penalization. Figure 3 illustrates the optimized thickness of a cantilever plate subject to: Allowable mass fraction is 50%; the bottom layer is enforced full density; intermediate void is not allowed; besides void there must be fiber angle continuity in an entire layer (MC1); total allowable thickness change is ± 1 ply thickness between neighbouring ESL shell elements (MC2); only two contiguous identical layers/plies are allowed (MC3). The result is completely binary, $x_{ijk} \in \{0,1\}$ and $\rho_{ij} \in \{0,1\}$. The fiber angle distribution from bottom to top, where $0°$ is along the x-axis and counter-clockwise is positive, is: $[0°, 0°, +45°, 0°, +45°, 0°, 0°]$.

CONCLUDING REMARKS

A novel method for simultaneous determination of optimum variable integer thickness and fiber orientation in a laminated composite subject to certain manufacturing constraints has been presented. The method is applicable for designs of composite structures with a fixed outer geometry but may also be used for structures with a fixed plane of symmetry. In the example, the optimized thickness and fiber orientation agree with the moment distribution in a cantilever plate. The sequential linear programming strategy with an adaptive move limit update scheme causes large variations in the required number of iterations until convergence. The non-convexity of the posed optimization problem entails the risk of sub-optimal local minima. Although the presented example results in a binary design, this is not always the case. Different settings for the adaptive update scheme and alternative solution approaches are elaborated in future work.

References

[1] Bendsøe M.P., Sigmund O.: Topology Optimization - Theory, Methods and Applications. Springer Verlag, Berlin Heidelberg, 2003.
[2] Ghiasi H., Kazem F., Pasini D., Lessard L.: Optimum stacking sequence design of composite materials part II: Variable stiffness design. *Composite Structures* **93**(1):1–13, 2010.
[3] Gill P.E., Murray W., Saunders M.A.: SNOPT: An SQP algorithm for large-scale constrained optimization. *SIAM Review* **47**(1):99–131, 2005.
[4] Stegmann J., Lund E.: Discrete material optimization of general composite shell structures. *International Journal for Numerical Methods in Engineering* **62**(14):2009–2027, 2005.
[5] Stolpe M., Svanberg K.: An alternative interpolation scheme for minimum compliance topology optimization. *Structural and Multidisciplinary Optimization* **22**(2):116–124, 2001.