TOPOLOGY OPTIMIZATION PROCEDURES FOR REINFORCED CONCRETE DESIGN

Oded Amir*[a)]
*Department of Mechanical Engineering, Technical University of Denmark, Kongens Lyngby, 2800, Denmark

Summary A family of novel topology optimization procedures for reinforced concrete structures is presented. The approach is based on incorporating nonlinear material modeling of concrete together with an embedded formulation for the representation of reinforcement. The distribution of concrete and reinforcement is determined simultaneously by combining continuum-based and truss-based topology optimization in a single computational procedure.

BACKGROUND

Topology optimization is emerging as a generic digital design tool that can be utilized in a wide range of engineering fields, spanning from nano-scale photonics up to furniture and buildings [11, 3, 12]. Despite its integration into the design process in various industries, optimal design had so far little impact on traditional structural engineering as practiced in the construction industry. Considering structural design with reinforced concrete, various challenges arise due to the difficulty in combining numerical optimization tools with accurate constitutive models. Furthermore, topology optimization procedures aimed at effective material distribution are typically based on continuum finite element modeling. In the context of reinforced concrete, this poses another challenge because of the two distinct physical scales involved: reinforcement is achieved by inserting a very small volumetric ratio of discrete steel bars into the continuum concrete.

This paper presents a family of novel computational procedures that are tailored specifically for the purpose of optimizing reinforced concrete designs. Examples of potential applications are: 1) Optimize the distribution of reinforcement based on a fully digital work flow, which is attractive in the design of complex concrete components for which strut-and-tie models are not easily realized; 2) Optimize the distribution of concrete so that weight is minimized, leading to a reduction in cement production which is considered a major contributor to CO_2 emissions; and 3) Enhance collaboration between architects and engineers during the conceptual design phase, where optimized designs can provide the basis for choosing the structural form, combining efficient engineering with appealing aesthetics.

Up to date, the main contribution of topology optimization for reinforced concrete design is its potential utilization as a tool for generating strut-and-tie models [7]. This idea was recently extended by considering different material properties in tension and compression [13]. Typically a continuum-based approach is employed but truss topology optimization was also suggested for the same purpose [9]. In contrast to the approach presented here, these contributions do not address the complete design problem: further interpretation is required in order to proceed with solving the strut-and-tie model and computing the amount of reinforcement. In various aspects, the current study is inspired by and related to recent developments by Kato and co-workers [5, 6] regarding optimization of fiber reinforced concrete. It differs mainly in the aim to obtain general layouts as opposed to sizing and shape optimization of predefined reinforcement layers. Furthermore, it offers a significant extension by addressing also the optimization of the concrete phase.

COMPUTATIONAL APPROACH

A key component in the presented topology optimization procedures is the employment of the embedded formulation for representation of reinforcement bars (rebars). The main idea is that the stiffness of each individual rebar, modeled by a 1-D element, is added to the stiffness of the surrounding concrete domain, modeled by continuum elements. It is assumed that the deformation of the bar is compatible to that of the hosting element. Therefore the strain in the bar is related to the element nodal displacements and its stiffness can be added at the element nodes. In continuum-based topology optimization, the design domain is typically discretized using a structured grid mesh with equilateral elements. In truss topology optimization, a so-called 'ground structure' is constructed such that it consists of all possible connectivities of bars within the design domain. These practices make the embedded formulation very attractive because the complete truss ground structure can be embedded into the continuum grid in a straightforward manner.

Another important component is the nonlinear constitutive model for plain concrete. In a preliminary study aimed solely at optimizing the reinforcement layout [1], a gradient-enhanced damage model [10] was utilized. Current work is focused on incorporating more realistic models which combine plasticity with either damage [8, 4] or fracture [2]. For the purpose of gradient-based optimization, sensitivity analysis is performed in complete consistency with the path-dependency and the nonlocal characteristics of the material model. The primal analysis is represented as a transient, nonlinear coupled problem, leading to a backwards-incremental adjoint sensitivity analysis procedure.

For reinforcement layout optimization, the goal is to distribute a given volume of rebars within a certain concrete domain so that the stiffest structure is obtained. Displacement control is used for the nonlinear finite element analysis so the objective is posed as maximization of the end-compliance. In other words, we seek to maximize the load-bearing capacity in the final equilibrium state corresponding to the desired value of the total prescribed displacement. For optimizing both the concrete phase as well as the reinforcement, the aim is to minimize the volume of concrete subject to a constraint

[a)] Corresponding author. E-mail: odam@mek.dtu.dk.

on the end-compliance. Design parametrization follows the well-established SIMP approach relating the mathematical design variables to the stiffnesses of rebars and concrete elements via interpolation functions.

EXAMPLES

Several demonstrative examples are hereby presented. The optimized rebar layout for a simply supported beam subject to a point load is shown in Figure 1. The layout is plotted on one symmetric half alongside the damage distribution. For the same problem setting, the optimized concrete and rebar distribution is presented in Figure 2. Black regions represent concrete, white regions represent void and blue lines represent rebars. Concrete occupies 66% of the design domain but the load-bearing capacity is 80% of the capacity of a full beam. Finally, the rebar layout for a 2-D corbel is shown in Figure 3. In all cases, reinforcement occupies 0.5% of the volume of the full concrete domain.

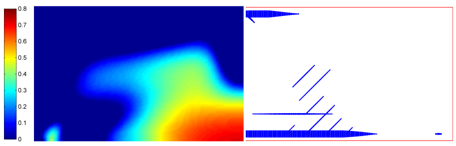

Figure 1. Damage distribution and optimized rebar layout, symmetric half of a beam with a point load.

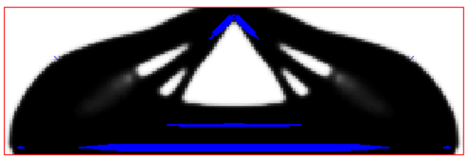

Figure 2. Optimized concrete and rebar distribution, beam with a point load.

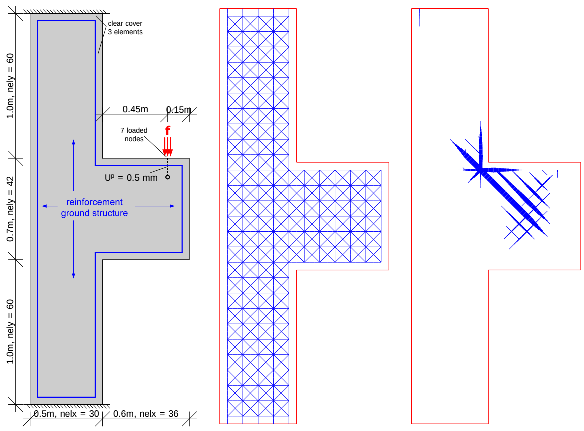

Figure 3. Reinforcement design for a 2-D corbel: problem setting, truss ground structure and optimized layout.

CONCLUSIONS

The developed framework, combining nonlinear material modeling of concrete with an embedded truss ground structure as reinforcement, provides the basis for optimizing the distribution of concrete and rebars simultaneously. In the resulting designs, rebars are positioned mainly in regions where concrete is severely damaged due to high tensile strains. Some reinforcement is used also in compression for strengthening in the vicinity of concentrated forces. An attractive aspect is the fully digital work flow, that can be very effective for designing complex 2-D and 3-D concrete structures.

References

[1] O. Amir and O. Sigmund. Reinforcement layout design for concrete structures based on continuum damage and truss topology optimization. Submitted.

[2] J. Červenka and V. K. Papanikolaou. Three dimensional combined fracture-plastic material model for concrete. *International Journal of Plasticity*, 24(12):2192 – 2220, 2008.

[3] P. Dombernowsky and A. Søndergaard. Three-dimensional topology optimisation in architectural and structural design of concrete structures. In *Proceedings of the International Association for Shell and Spatial Structures (IASS) Symposium*, Valencia, Spain, 2009.

[4] P. H. Feenstra and R. de Borst. A composite plasticity model for concrete. *International Journal of Solids and Structures*, 33:707–730, 1996.

[5] J. Kato, A. Lipka, and E. Ramm. Multiphase material optimization for fiber reinforced composites with strain softening. *Structural and Multidisciplinary Optimization*, 39(1):63–81, 2009.

[6] J. Kato and E. Ramm. Optimization of fiber geometry for fiber reinforced composites considering damage. *Finite Elements in Analysis and Design*, 46(5):401–415, 2010.

[7] Q. Liang, Y. Xie, and G. Steven. Topology optimization of strut-and-tie models in reinforced concrete structures using an evolutionary procedure. *ACI Structural journal*, 97(2):322–330, 2000.

[8] J. Lubliner, J. Oliver, S. Oller, and E. Oñate. A plastic-damage model for concrete. *International Journal of Solids and Structures*, 25:299–326, 1989.

[9] C. D. Moen and J. K. Guest. Reinforced concrete analysis and design with truss topology optimization. In *Proceedings of the 3rd fib International Congress*, Washington D.C., USA, 2010.

[10] R. H. J. Peerlings, R. de Borst, W. A. M. Brekelmans, and J. H. P. de Vree. Gradient enhanced damage for quasi-brittle materials. *International Journal for Numerical Methods in Engineering*, 39(19):3391–3403, 1996.

[11] O. Sigmund and M. P. Bendsøe. Topology optimization: from airplanes to nano-optics. In K. Stubkjær and T. Kortenbach, editors, *Bridging from technology to society*. Technical University of Denmark, Lyngby, Denmark, 2004.

[12] L. L. Stromberg, A. Beghini, W. F. Baker, and G. H. Paulino. Application of layout and topology optimization using pattern gradation for the conceptual design of buildings. *Structural and Multidisciplinary Optimization*, 43:165–180, 2011.

[13] M. Victoria, O. M. Querin, and P. Martí. Generation of strut-and-tie models by topology optimization using different material properties in tension and compression. *Structural and Multidisciplinary Optimization*, 44:247–258, 2011.