NUMERICAL PERFORMANCE OF CHAOS OPTIMIZATION ALGORITHMS BASED ON DIFFERENT MAPS FOR GLOBAL OPTIMIZATION

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Summary The computational performance of hybrid chaos-BFGS optimization algorithms is compared based on different chaotic maps. Numerical results of typical nonlinear multimodal functions illustrate that, the global optimization efficiency of hybrid chaos optimization algorithms is significantly affected by the probability distribution property and ergodic velocity (characterized by Lyapunov exponent) of chaotic time series of chaotic maps.

INTRODUCTION

Many problems in various kinds of fields can be formulated as a typical optimization model, and it is important to search for the global optimum for most of them. However, the global optimum is not characterized by any mathematical condition, although the local optimum satisfies a necessary condition of KKT system. The global optimization of nonlinear, non-convex and non-differential problems is still an open great challenge for researchers. In recent years, the theories and applications of chaotic dynamics have drawn more and more attention in the fields of science and engineering. One field is the potential application of chaos in optimization. Nevertheless, the chaos optimization algorithm (COA) needs to spend much computational effort to approach the global optimum eventually by searching numerous points. Hence, the hybrid methods were developed, in which the chaos optimization algorithm was used for global search and the conventional optimization algorithms (e.g. BFGS algorithm) were employed for local search near the global optimum. It can save much CPU time and enhance the computational efficiency of algorithms [1].

Yang *et al*. [1] demonstrated the remarkable influence of probability density functions (PDF) of chaotic time series of three iterative maps on the global optimization capacity of hybrid chaos-BFGS optimization algorithm. Tavazoei *et al*. [2] proposed the weighted gradient direction based chaos optimization algorithm, and compared numerical simulation results using ten different one-dimensional maps as chaotic search patterns in the constraint nonlinear optimization problems. However, they did not consider whether the characteristics of chaotic maps affect the numerical performance of chaos optimization algorithm.

In this paper, we choose seven representative one-dimensional chaotic maps to form the hybrid chaos-BFGS optimization algorithm, and expose the effect of statistical property (measured by PDF) and ergodic rate (characterized by Lyapunov exponent) of the chaotic time series of different chaotic maps on the efficiency of hybrid chaos optimization algorithm.

HYBRID CHAOS-BFGS OPTIMIZATION ALGORITHM

Consider the optimization problem for nonlinear multimodal function with boundary constraints:

$$\text{Minimize} \quad f(\mathbf{x}) = f(x_1, x_2, \ldots, x_n) \tag{1a}$$

$$\text{Subject to} \quad XL_i \le x_i \le XU_i \quad (i=1,2,\ldots,n) \tag{1b}$$

where f denotes objective function with respect to n design variables x_i (i=1,2,…,n) with the assumption of continuously twice differentiability. XL_i and XU_i are lower and upper bounds for variable x_i. In order to generate the design variable by chaotic map of chaos optimization algorithms [1], a linear mapping between the design variable x_i and chaos variable z_i is defined, namely,

$$z_i = (x_i - XL_i)/(XU_i - XL_i) \tag{2}$$

The chaos optimization algorithm sometimes can obtain global optimum, but the effectiveness and efficiency is not good due to the limited local optimization ability of chaotic search. The conventional gradient descent algorithms are good at local optimization. By integrating these two algorithms we can develop a more efficient algorithm for global optimization. Because BFGS algorithm is one of robust and efficient quasi-Newton algorithms of local optimization, it is selected to combine with chaos optimization algorithm to form the hybrid global optimization algorithm [1].

NUMERICAL COMPARISON OF HYBRID CHAOS-BFGS GLOBAL OPTIMIZATION

The efficiency of the above hybrid chaos global optimization algorithms with different one-dimensional chaotic maps [1-5] is evaluated on the basis of the following several nonlinear multimodal functions.

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$$f_1(x,y)=(4-2.1x^2+\frac{x^4}{3})x^2+xy+(-4+4y^2)y^2 \qquad -10<x,y<10 \tag{3}$$

$$f_2(x,y)=0.5-\frac{\sin^2\sqrt{x^2+y^2}-0.5}{(1+0.001(x^2+y^2))^2} \qquad -4<x,y<4 \tag{4}$$

$$f_3(\mathbf{x})=\sum_{i=1}^{3}(x_i^2-10\cos(2\pi x_i)+10) \qquad -4<x_i<4, i=1,2,3 \tag{5}$$

$$f_4(\mathbf{x})=\frac{1}{4000}\sum_{i=1}^{5}x_i^2-\prod_{i=1}^{5}\cos(\frac{x_i}{\sqrt{i}})+1 \qquad -5<x_i<5, i=1,2,\ldots,5 \tag{6}$$

$$f_5(\mathbf{x})=\frac{1}{5}\sum_{i=1}^{5}(x_i^4-16x_i^2+5x_i) \qquad -10<x_i<10, i=1,2,\ldots,5 \tag{7}$$

One-dimensional noninvertible maps are the simplest systems with capability of generating chaotic motion [5]. The well-known one-dimensional maps [2] include the Logistic map, Kent map, Sine map, Circle map, Bernoulli shift map, ICMIC map (The iterative chaotic map with infinite collapses), and Chebyshev map. Some of them are listed as follows:

Logistic map $$z_{n+1}=\mu z_n(1-z_n) \qquad z\in[0,1] \qquad 0<\mu\le 4 \tag{8}$$

Sine map $$z_{n+1}=\frac{a}{4}\sin(\pi z_n) \qquad z\in[0,1] \qquad 0<a\le 4 \tag{9}$$

Table 1. Probability searching for global optimum by hybrid COA with different maps for multimodal functions (%)

LE	map	f_1			f_2			f_3			f_4			f_5		
		100	500	1000	1000	5000	10^4	10^4	4×10^4	10^5	10^4	4×10^4	10^5	10^4	4×10^4	10^5
0.4336	Circle	60	77	74	100	100	100	100	100	100	96	100	100	100	100	100
0.4339	Sine	79	99	100	60	92	100	70	94	100	25	28	27	48	70	90
0.4330	Kent	72	97	100	53	95	100	24	39	59	24	25	27	32	45	58
0.4330	Bernoul	76	98	100	56	94	100	24	39	62	22	27	31	33	50	58
0.4335	Logistic	69	94	99	44	58	81	6	16	25	8	10	12	22	25	23
0.4273	ICMIC	30	43	56	23	35	49	1	2	3	1	1	2	15	17	17
0.4337	Chebysh	0	0	0	0	0	0	0	0	0	0	0	0	1	1	2

Table 1 shows the probability searching for global optimum by hybrid COA based on different chaotic maps with the close Lyapunov exponents (LE) and different PDF of chaotic sequences. It is illustrated that the probability distribution of chaotic series of chaotic maps significantly influences the efficiency of chaos optimization algorithms.

Table 2. Probability searching for global optimum by hybrid COA with Kent map for multimodal functions (%)

cases		f_1			f_2			f_3			f_4			f_5		
		100	500	1000	1000	5000	10^4	10^4	4×10^4	10^5	10^4	4×10^4	10^5	10^4	4×10^4	10^5
β=0.05	LE=0.1985	44	78	88	13	43	60	7	21	27	9	11	15	22	32	46
β=0.1	LE=0.3251	58	87	98	22	75	93	18	30	58	12	16	19	26	35	49
β=0.2	LE=0.5004	72	96	100	46	91	100	20	38	60	15	22	25	29	40	52
β=0.3	LE=0.6109	75	98	100	52	93	100	22	44	60	20	27	29	29	40	54
β=0.4	LE=0.6730	76	98	100	62	99	100	23	44	67	23	27	30	30	40	55

Table 2 displays the efficiency of hybrid COA based on Kent map with the same PDF and different Lyapunov exponent. Numerical results indicate that the larger Lyapunov exponent of chaotic series corresponds to the higher probability searching for global optimum of multimodal functions by hybrid chaos optimizations algorithms. Meanwhile, the chaos optimization algorithm is extended to solve the global optimization of engineering problems with various constraints.

CONCLUSIONS

Several multimodal functions are tested to compare the performance of hybrid chaos-BFGS optimization algorithms with different chaotic maps. It is demonstrated that the PDF and Lyapunov exponent of chaotic series of chaotic maps play important role in searching for the global optimum by hybrid chaos optimization algorithms. Moreover, the chaotic series of iterative map with uniform distribution and large Lyapunov exponent is beneficial to the global optimization.

References

[1] Yang D.X., Li G., Cheng G.D.: On the efficiency of chaos optimization algorithms for global optimization. *Chaos,Solitons and Fractals* **34(4)**:1366-1375, 2007.
[2] Tavazoei M.S., Haeri M.: Comparison of different one-dimensional maps as chaotic search pattern in chaos optimization algorithms. *Applied Mathematics and Computation* **187**:1076-1085, 2007.
[3] Li B., Jiang S.W.: Optimizing complex function by chaos search. *Cybernetics and Systems* **29(4)**: 409-419, 1998.
[4] Zhang T., Wang H.W., Wang Z.C.: Mutative scale chaos optimization algorithm and its application. *Control and Decision* **14(3)**:285-288, 1999.
[5] Ott E.: Chaos in Dynamical Systems. Cambridge University Press, Cambridge 2002.