RECTIFICATION OF CLASSICAL MISCONCEPTIONS IN RECENT SOLUTIONS OF EXACT STRUCTURAL TOPOLOGY OPTIMIZATION PROBLEMS

George Rozvany* & Tomasz Sokół **
*Department of Structural Mechanics, Budapest University of Technology and Economics, Budapest 1521, Hungary
** Faculty of Civil Engineering, Warsaw University of Technology, Al. Armii Ludowej 16, 00-637 Warsaw, Poland

Summary This paper deals with exact analytical solutions of topology optimization problems for (Michell) trusses with one loading condition and a stress constraint. New classes of solutions are presented, which have important general implications for fundamental properties of optimal topologies. In particular, solutions are investigated, for which Michell's original optimality criteria, or Hemp's orthogonality principles are not valid. Moreover, the 'principle of local violations' is discussed, and solutions are presented with previously rarely used 'optimal regions'. All analytical solutions are verified numerically using over hundred million potential members in some calculations.

SHORTCOMINGS IN MICHELL'S TRUSS THEORY

Michell's (1904) milestone contribution to truss topology optimization is undoubtedly both revolutionary and ingenious. It was therefore certainly not the first author's intention to belittle Michell's immense effort when he pointed out (Rozvany 1996) some limitations of Michell's (1904) original optimality criteria. These limitations included (i) equal permissible stresses in tension and compression, or alternatively 'statically definite' support conditions (allowing uniform dilatation). The first author (Rozvany 1996) also derived by three different methods modified optimality criteria, which agreed with those proposed earlier by Hemp (1973). He also showed on a simple example that the modified optimality criteria result in a lower truss volume than those derived from conditions of Michell. It is clear from Michell's (1904) paper that he mistakenly believed that his criteria are also valid for unequal permissible stresses.

RELAXATION OF HEMP'S ORTHOGONALITY PRINCIPLE

In his most invaluable book, Hemp (1973) stated that in Michell trusses, a compression and a tension member meeting at a point must be orthogonal. This principle is certainly valid for so-called T-regions, which were considered in Hemp's solutions. It was pointed out by the first author (Rozvany 1997) that a compression bar and a tension bar may not be orthogonal in optimal Michell topologies, if they meet at the boundary of a so-called R+ and an R- region. In the quoted paper, only symmetric examples (single point load, two hinge supports, two symmetric bars) were shown. More recently the first author derived also non-symmetric examples of non-orthogonal bars in optimal Michell topologies, and these were verified numerically by the second author, who used more than hundred million potential bars in some of his calculations. One analytical solution of the first author is shown in Fig. 1.

EXTENSIONS OF MICHELL'S BEST KNOWN SOLUTION

Figure 2b shows a solution, which is often attributed to Michell (1904), although his solution was a truss symmetric about a horizontal axis, with two given reaction forces. The authors extended Michell's solution to eight different cases, with (a) equal or unequal permissible stresses in tension and compression, (b) the structural domain consisting of a half-plane or a full plane, and (c) the supports comprising two pins or a pin and a roller. The correct solution for unequal permissible stresses, pin supports and a half-plane is shown in Fig. 2a. For this problem Michell's original criteria would give the solution in Fig. 2b, which has been found to have, as expected, a higher volume.

EXTENSIONS OF MICHELL'S SOLUTION TO TWO POINT LOADS

Michell's classical solution (full plane equivalent of Fig. 2b) was extended to two point loads by the authors. *A rather astonishing historical aspect of this development is the fact that it has taken over hundred years to find the two-load extension of Michell's best-known solution with one load.* The half-plane versions of these two topologies were derived earlier by Sokol and Lewinski (2010).

MICHELL'S SOLUTION WITH ALLOWANCE FOR COST OF SUPPORTS TO TWO POINT LOADS

The theory of Michell trusses with nonzero support cost has also been formulated and applied to particular support and load conditions. For a single central point load the optimal topology has turned out to be the same as the one shown in Fig. 2a.

ADJOINT STRAIN FIELDS FOR REGIONS WITHOUT MEMBERS

Melchers (2005) noted the difficulties in constructing exact optimal adjoint strain fields for regions without members (termed O-regions). For such regions numerical solutions help also very little. However, the authors have now succeeded in deriving exact O-regions, using sub-regions termed Z-regions and V-regions.

THE PRINCIPLE OF LOCAL VIOLATIONS

This principle was originally demonstrated in the context of optimal grillage topologies (Rozvany 1994), and also confirmed numerically by Sigmund et al (1993). It states that over an infinitesimal region of an optimal region we may violate the optimality criteria. Now it is applied in deriving Michell trusses.

CONCLUDING REMARKS

The above developments and their important implications will be explained in detail in the lecture.

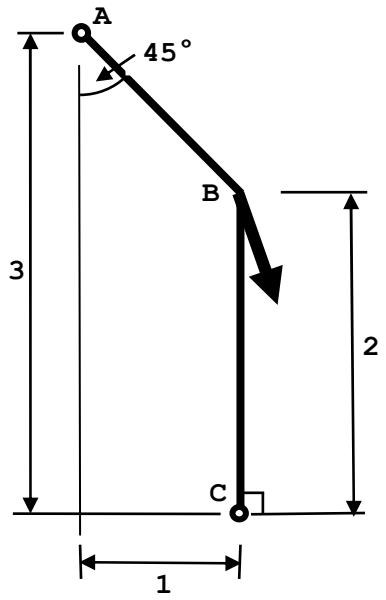

Fig.1. Optimal topology violating Hemp's orthogonality principle. Bar AB is in tension, bar BC in comppression.

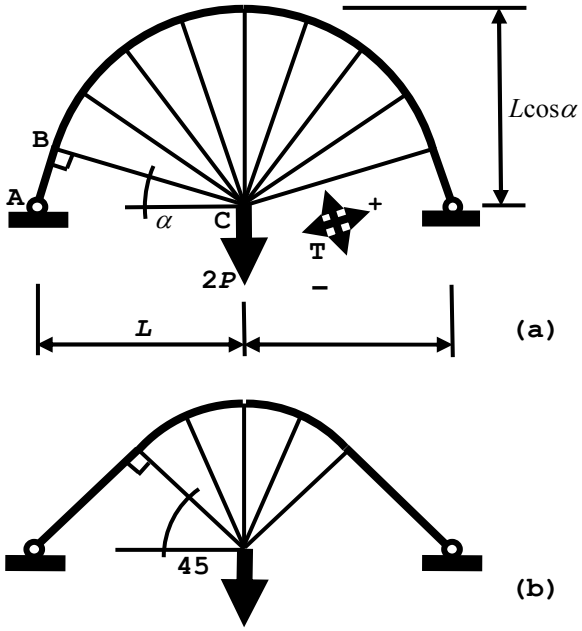

Fig.2. (a) The correct optimal topology for unequal permissible stresses in tension and compression, (b) modified Michell's solution.

References

Hemp WS (1973) Optimum structures. Clarendon, Oxford

Melchers R (2005) On extending the range of Michell-like optimal topology structures. Struct Multidisc Optim 29:85-92

Michell AGM (1904) The limits of economy of material in frame structures. Phil Mag 8:589-597

Rozvany GIN (1994) Topological optimization of grillages: past controversies and new directions. Int J Mech Sci 36:495-512

Rozvany GIN (1996) Some shortcomings of Michell's truss theory. Struct Optim 12:244-250

Rozvany GIN (1997) Partial relaxation of the orthogonality requirement for classical Michell trusses. Struct Optim 13:271-274

Sigmund O, Zhou M, Rozvany GIN (1993) Layout optimization of large FE-systems by new optimality criteria methods: applications to beam systems. In: Haug EJ (ed) Proc NATO ASI: Concurrent Engineering tools and technologies for mechanical system design (Iowa, 1992) Springer-Verlag, Berlin

Sokół T, Lewiński T (2010) On the solution of the three forces problem and its application to optimally designing a class of symmetric frameworks of least weight. Struct Multidisc Optim, 42:835-853