OPTIMIZATION OF PRAGER STRUCTURES BY FINITE ELEMENT METHOD

Kemin Zhou [a)], Ronglin Chen
College of Civil Engineering, Huaqiao University, Fujian, 361021, China

Summary Prager structures are optimized by finite element method. Spatial truss-like continuum material model is established. It is assumed that, at any point in the truss-like material model, there are three families of members along three orthotropic orientations. The members are distributed non-uniformly continuously. The densities and orientations of members at nodes are taken as design variables. The densities of members are optimized by fully stressed criterion. The orientations of members are aligned to principal stress orientations. The positions of loads point along vertical direction are changed in iteration. The optimal structural shapes are determined by the center of mass. Several numerical examples are provided to illustrate the effectiveness of this method.

INTRODUCTION

According to the geometric characters of optimized structures, optimized structures are divided as plane, space, plate and shell structures. Prager structures are is a kind of special shell structures, in which its outline is surface, which covers a region in a horizontal plane. Structures are supported at its boundaries. Loads act along vertical direction. Their horizontal positions are prescribed and heights are not prescribed. These least weight structures with stress constraints are called "archgrid-like" or "Prager structure [1-3]. The "archgrid" is similar to "truss-like" and "grillages-like". It means that members are distributed non-uniformly continuously. Their densities and orientations vary continuously in the design domain in a shell.

Rozvany and Prager studied the optimal criterion of least weight shell with stress constraint. On the condition that orientations of members were prescribed in two vertical orientations, the iterative difference method was established. However the orientations of members are not optimized. Although similar space structures are optimized by various methods with finite element analysis, Prager structures are load-depend problem, it is different from other optimization problem greatly.

SPATIAL TRUSS-LIKE CONTINUA MATERIAL MODELS

Elastic matrix of the truss-like material model

In truss-like material model, there are three families of members along three orthotropic orientations. The three orientations of members are named as material principal axis, denoted as 1, 2 and 3. The cross-sectional areas of members in per unit area are defined as densities of members, denoting as t_1, t_2 and t_3 along three orthotropic orientations, respectively. The elastic matrix along material principal axis is assumed as,

$$\boldsymbol{D}(\boldsymbol{t},0)=E\cdot\mathrm{diag}[t_1\quad t_2\quad t_3\quad (t_2+t_3)/4\quad (t_3+t_1)/4\quad (t_1+t_2)/4], \tag{1}$$

where diag is diagonal matrix. The last three terms in the matrix in (1) are shear stiffness, which is assumed to avoid stiffness matrix becoming singular. Particularly, on the condition of t_1=t_2=t_3, the material becomes isotropic.

The densities $\boldsymbol{t}_j=\begin{bmatrix}t_{1j} & t_{2j} & t_{3j}\end{bmatrix}^{\mathrm{T}}$ and orientations $\boldsymbol{n}_{bj}=\begin{bmatrix}l_{bj} & m_{bj} & n_{bj}\end{bmatrix}^{\mathrm{T}}$ of members at nodes are taken as design variables, where l_{bj}, m_{bj}, n_{bj} are directional cosine of b member at node j. The elastic matrix at node j is denoted as,

$$\boldsymbol{D}_j=\boldsymbol{D}(\boldsymbol{t}_j,\boldsymbol{n}_{bj})=E\sum_{b=1}^{3}t_{bj}\sum_{r=1}^{6}g_r(\boldsymbol{n}_{bj})\boldsymbol{A}_r\ , \tag{2}$$

where S_e is the set of nodes belong to element e, $\boldsymbol{A}_r$ six of constant matrixes,

$\boldsymbol{A}_1$=diag[4,0,0,0,1,1], $\boldsymbol{A}_2$=diag[0,4,0,1,0,1], $\boldsymbol{A}_3$=diag[0,0,4,1,1,0],
$\boldsymbol{A}_4$=[0,0,0,0,0,0; 0,0,0,2,0,0; 0,0,0,2,0,0; 0,2,2,0,0,0; 0,0,0,0,0,1; 0,0,0,0,1,0],
$\boldsymbol{A}_5$=[0,0,0,0,2,0; 0,0,0,0,0,0; 0,0,0,0,2,0; 0,0,0,0,0,1; 2,0,2,0,0,0; 0,0,0,1,0,0],
$\boldsymbol{A}_6$=[0,0,0,0,0,2; 0,0,0,0,0,2; 0,0,0,0,0,0; 0,0,0,0,1,0; 0,0,0,1,0,0; 2,2,0,0,0,0]

g_r the components of vector function of orientations,

$$\boldsymbol{g}(\boldsymbol{n}_b)=[l_b^2\quad m_b^2\quad n_b^2\quad m_bn_b\quad n_bl_b\quad l_bm_b]\ , \tag{3}$$

Stiffness matrix

Stiffness matrix is calculated by,

$$\boldsymbol{K}=\sum_{e}\sum_{j\in S_e}\sum_{b=1}^{3}t_{bj}\sum_{r=1}^{6}g_r(\boldsymbol{n}_{bj})\boldsymbol{H}_{erj}\ , \tag{4}$$

where S_e is the set of nodes belong to element e, $\boldsymbol{H}_{erj}$ a constant matrix for regular elements,

$$\boldsymbol{H}_{erj}=E\int_{V_e}N_j\boldsymbol{B}^{\mathrm{T}}\boldsymbol{A}_r\boldsymbol{B}\mathrm{d}V\ , \tag{5}$$

[a)] Corresponding author. Email: Zhoukm@hqu.edu.cn

OPTIMIZATION APPROACH

Optimization problem

The optimization problem is stated as,

$$\begin{array}{l} \text{find } t_{bj}, \boldsymbol{n}_{bj} \\ \min V \\ \text{s.t. } \sigma_{bj} \le \sigma_0 \end{array}, \quad \begin{array}{l} b = 1, 2, 3; \\ j = 1, 2, \ldots, J \end{array}, \tag{6}$$

where σ_{bj} is stress of member b at node j, σ_0 prescribed permitted stress, V the material volume. For densities varying in element, the material volume at any point in element can be calculated by integration of densities of members at nodes,

In Prager structures, loads act on structures along vertical directions. For structures being not prescribed in optimization process, the vertical positions of loads are not prescribed. However, structures can not be analyzed before positions of loads are prescribed. To overcome this difficulty, loads are assumed to distribute uniformly initially and rearranged along vertical direction according to the changes of densities of members in iteration.

Iteration procedure

(1) The magnitudes and orientations of principal stresses at nodes are calculated by finite element analysis;

(2) Members are aligned to principal stress direction. The densities of members are updated by fully stressed criterion;

Iteration (1) and (2) is repeated until the changes of design variables is little enough, $<10^{-4}$ in this paper, for example. The topology optimization structures are formed by linking the centroid of members.

NUMERICAL EXAMPLES

A cubic domain under uniform distributed loads with four edges fixed in bottom, as shown in Figure 1. For symmetry, only quarter of structure is analyzed. Cubic elements of 12×12×12 are used. Shell is optimized after iteration of 13. The optimized cross sections are shown in Figure 2. These lines from up to down represent the parallel sections of shell

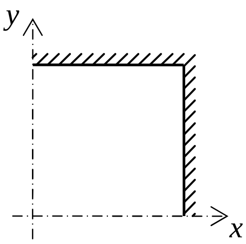

Figure 1 Mechanics model

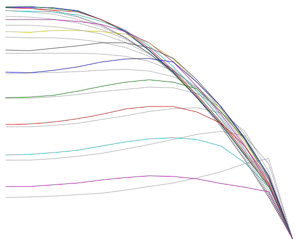

Figure 2 Comparison of sections with Ref [1]

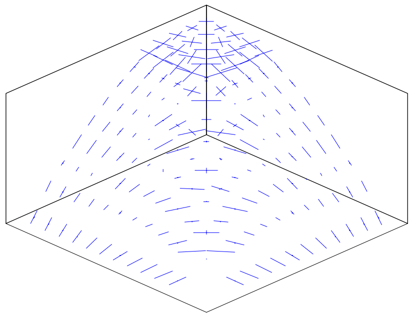

Figure 3 Optimized structure

from far to near to x-axis. Thin dot-line is the solution obtained by difference method [1]. The error of height and volume is 4.2% and 4.8% relative to difference solution, respectively. The densities and orientations of members on curve surface are shown in the axonometric drawing in Figure 3. The orientations and length of lines stand for the orientations and densities of members. It is shown from the Figure 3 that these members are not parallel or vertical for each other completely. The most loads near the coordinate axes are carried by two orthotropic arches. The loads near the corners are carried by the small arches of 45° crossing with coordinate axis.

CONCLUSIONS

A method to optimize Prager structures based on truss-like continuum material model is presented. Not only the structural shape but also the orientations of members are optimized. The orientations may vary in the design domain continuously, rather than keeping constant.

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