SOME INVESTIGATION ON THE SHAPE OPTIMIZATION OF COMPLEX NONLINEAR STRUCTURES

Zhenhan Yao[*a)] & Yingtao Wei [**]
[*]*Department of Engineering Mechanics, Tsinghua University, Beijing, 100084, China*
[**]*Department of Automotive Engineering, Tsinghua University, Beijing 100084, China*

Summary For the complex nonlinear structures, FE analysis for each design sample is time-consuming; therefore, how to get the optimal design with fewer samples is very important. The gradient-based Kriging method based on the samples determined by the orthogonal maximin Latin hypercube design is adopted in the investigation. A new criterion for the best likelihood spatial correlation parameter is presented and verified using a series of test examples. For the sensitivity analysis of nonlinear structure with respect to the shape variation, it is formulated based on a presented geometrical mapping approach, rather than the material derivative approach in literature. Started from the virtual work principle, the sensitivity equations of state variables with respect to the shape variation and the equation for calculating the sensitivity of performances have been formulated. To enhance the efficiency, a new method to determine the domain design velocity field is presented.

INTRODUCTION

Structural optimization is one of the important tasks in modern structural design, and it is usually based on the FE analysis and optimization algorithms. For the complex nonlinear structures, such as automotive tires, FE analysis for each design sample is time-consuming; therefore, how to get the optimal design with fewer samples is very important. To reduce the number of samples, the information gained from each sample should be enhanced, by using sensitivity analysis; correspondingly the gradient-based Kriging (GBK) method can be adopted as the optimization algorithm.

SOME INVESTIGATION ON THE GBK METHOD

For the GBK method based on the samples determined by the orthogonal maximin Latin hypercube design, a criterion for the best likelihood spatial correlation parameter is presented. This criterion is based on the transition from insufficient to excessive control of the Kriging surface via spatial correlation parameter. The criterion is verified using a series of test examples. If the response functions are power function, or trigonometric function sine, cosine in the range of π without random error, the sample number greater than 1.5 times of the number of design variables is enough. If the response functions are sine or cosine functions in the range of 2π without random error, the sample number should be greater than 3 times of the number of design variables. The test examples have shown that with random error less than 10%, the GBK method can still be applied, sometimes more samples are required. Actually the treated response function and the optimization results are also approximated in such cases. Therefore, to ensure the accuracy of the FE analysis and sensitivity analysis is very important.

SENSITIVITY ANALYSIS OF NONLINEAR STRUCTURE WITH RESPECT TO SHAPE VARIATION

Geometrical Mapping Approach, rather than the Material Derivative Approach

For the sensitivity analysis of nonlinear structure with respect to the shape variation, it is formulated based on the geometrical mapping approach (GMA), rather than the material derivative approach. The material derivative approach presented in literature has some concept confusion, because the real material derivative is not suitable to formulate the variation of shape design. The partial derivative with respect to the shape parameter did not have any physical meaning, and the gradient there is the derivative with respect the Euler coordinates rather than the Lagrange coordinates, which appears in the displacement gradient of the solids. The GMA is presented with clearer physical meaning, wherein the shape variation is regarded as a mapping characterized by the shape variation velocity field, or a fictitious deformation of the continuum, which is not real displacement, and there are no strain and stress corresponding to such fictitious displacement field.

Derivation of the Sensitivity Equations

Started from the virtual work principle for modified shape and then mapping to the original shape, the sensitivity equations of state variables for nonlinear structure with respect to the shape variation have been formulated. Corresponding to the sensitivity of displacements the matrix of the equation system is identical with conventional nonlinear structural FE analysis, namely the tangential stiffness matrix including the linear, nonlinear and geometrical stiffness matrices. On the right hand as the load terms, there are the fictitious loads resulted by shape variation, the rate

[a)] Corresponding author. Email: demyzh@tsinghua.edu.cn.

of external load, the rate of contact loads, and the term related to the sensitivity of Lagrange multiplier for contact problem. Besides this equation system, there are additional contact constraint equations. All terms in the fictitious loads resulted by shape variation are related to the design velocity field. Finally, the equation for calculating the sensitivity of performances with respect to the shape variation, which will be applied in the shape optimization algorithm such as GBK method, is also derived.

A New Method to Determine the Domain Design Velocity Field

To enhance the efficiency, a new method to determine the domain design velocity field is presented. After the boundary design velocity field formulated via design parameters, the domain velocity field can be computed by a lot of methods in literature, such as the boundary displacement method, the fictitious load method and others. The resulted domain velocity field is almost distributed in whole domain and the computation takes time in general. In the presented new method to determine the domain velocity field, the domain velocity field is localized in one layer of finite elements, which directly related to the design modified boundary. For HEX8 elements, domain velocity field in element is directly determined by the boundary velocity field and no additional computing cost is required. Furthermore, the undesirable mesh distortion for composite structure can be avoided naturally.

Adjoint Variable Method

To enhance the efficiency of sensitivity analysis, the adjoint variable method is applied, wherein the asymmetry of the equation matrix due to the slip contacted case of the frictional contact using Lagrange multiplier method is taken into account.

To Ensure the Accuracy of Sensitivity is Very Important for the Shape Optimization

For the shape optimization of complex nonlinear structures, it is very important to ensure the accuracy of sensitivity. Therefore, the equations are accurately derived and for contact problem the Lagrange multiplier approach is applied. Furthermore, the reasonable mesh should be adopted for each shape design sample, different mesh for different shape is reasonable. The design velocity field should also be determined for each shape design sample, corresponding to small variation of design parameter. Otherwise the optimization will lose the reliability.

CONCLUDING REMARKS

Two of the key issues for the shape optimization of complex nonlinear structure are presented in this paper. One is for the GBK method, a criterion for the determination of maximum likelihood spatial correlation parameter is presented, which is based on the samples determined by the orthogonal maximin Latin hypercube design, and searching for the transition from insufficient control to successive control. This criterion has been verified by a series of test examples. The second is for the sensitivity analysis with respect to the shape design, the GMA is presented, which has clearer physical meaning than so called material derivative approach. On this basis the equation system for the sensitivity of displacement is derived from the virtual work equation, and then the sensitivity of performance can be obtained by the adjoint variable method.

The shape optimization of complex nonlinear structure should be based on the FE and sensitivity analysis, or numerical experiments. The physical experiments are characterized with random error, which usually can be reduced significantly by repeated measurements. The numerical experiments also have error, including the error of modeling, the discretization error and calculating error, which cannot be reduced by repeated computation. On the other hand, the numerical examples using GBK shows that although it is very efficient for the observation data of accurate response function, it will be inefficient for the data with considerable error. Only if the numerical error was limited in an admissible range, the optimization using GBK would be efficient, sometimes with more design samples than for the case without error. Therefore, to ensure the accuracy of the FE and sensitivity analysis is very important. First of all is to ensure the accuracy of the formulation. Therefore the GMA is applied, and the Lagrange multiplier approach is applied for the contact analysis. Furthermore, the reasonable mesh should be adopted for each shape design sample, different mesh for different shape is reasonable; design velocity field should also be determined for each shape design sample, corresponding to small variation of the design parameter. Otherwise the shape optimization will lose the reliability.

More details on the sensitivity analysis by GMA and the optimization using GBK method with the presented criterion for determination of the maximum likelihood spatial correlation parameter, and the practical engineering applications, will be presented in separated papers.

References

[1] Xuan Y, Xiang JH, Zhang WH, Zhang YL. Gradient-based Kriging approximate model and its application research to optimization design. *Sci China E: Tech Sci*, **52**: 1117–1124, 2009.

[2] Choi KK, Kim NH. *Structural Sensitivity Analysis and Optimization 1: Linear Systems*. New York: Springer; 2005.

[3] Choi KK, Kim NH. *Structural Sensitivity Analysis and Optimization 2: Nonlinear Systems and Applications*. New York: Springer; 2005.

[4] Joseph VR, Hung Y. Orthogonal-maximin Latin hypercube designs. *Statistica Sinica*, **18**: 171-186, 2008.