HYSTERETIC BEHAVIOR OF OCCURRENCE OF STRAIN NONUNIFORMITY IN THIN-WALLED CYLINDERS SUBJECTED TO CYCLIC TORSION, ANALOGOUS TO INTERNAL BUCKLING

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Summary Numerical analysis for the occurrence of strain nonuniformity in elastic-plastic thin-walled cylinders subjected to cyclic torsion is carried out, which is caused by in-plane branching behavior analogous to *internal buckling* shown by Biot. From hysteretic behavior analysis of the cylinder with a small imperfection it is shown that rapid growth of strain nonuniformity occurs and various deformation modes appear for some different shapes of imperfection. Also branching analysis for the same model is performed according to *symmetry limit theory* and the predicted multiple branching behavior is compared with the hysteretic behavior.

INTRODUCTION

The occurrence of nonuniform strain was observed in an experimental study concerning steel circular tubes subjected to cyclic torsion [2]. This type of experiments is frequently conducted to investigate multiaxial material behaviors on the assumption of uniform strain. Such nonuniformity is inconvenient and needs to be prevented. In a previous study [5], the authors applied *symmetry limit theory* [4], for predicting bifurcation phenomena under cyclic loading, to this problem, and showed that the short-wave deformation mode causing strain nonuniformity could occur in a cylinder subjected to cyclic torsion, analogous to the *internal buckling* of a rigidly confined continuum shown by Biot [1]. In this study, hysteretic behavior numerical analysis of an elastic-plastic thin-walled cylinder under cyclic torsion will be carried out for the verification of the previous theoretical prediction and the further clarification of this phenomenon. The results will be compared with the previously predicted multiple branching behaviour, and will reveal very rapid growth of the strain nonuniformity and the appearance of various deformation modes.

HYSTERETIC BEHAVIOR ANALYSIS

Consider an elastic-plastic thin-walled cylinder, which is cut and opened along the longitudinal line and is regarded as a 2-D continuum with a periodic boundary, shown in Fig. 1. Finite element discretization is applied. Constraining the upper end to deform uniformly, a constant axial compressive force is applied. For the constraint of the upper end two patterns of small imperfection are considered; in case (i), four nodal points which are not adjacent, are constrained 0.005mm lower than others, and in case (ii), only one nodal point is constrained 0.005mm lower than others. Then a completely reversed cyclic torsional displacement is applied, in which amplitude ψ increases every half cycle. The present critical behavior is supposed to be highly nonlinear and incremental perturbation method [3] effective for such analysis is used, in which the Taylor's expansion is taken to the degree of five.
For axially adjacent four elements A to D shown in Fig.1, the variations of the axial strain and the shear strain for cases (i) and (ii) are shown in Figs. 2(a) and 2(b), respectively. Within the elastic range the strain distribution is almost uniform and yielding occurs in the whole region around ψ = 0.0018. From around ψ = 0.0020 the difference in the shear strain among four elements increases and grows large. Also from around ψ = 0.0021 the difference in the axial strain increases. In particular the shear strain distributions are very different between cases (i) and (ii). On the other hand, for circumferentially adjacent eight elements 1 to 8 shown in Fig.1, such increase of the difference in these strains is not observed. Also numerical analysis under monotonic torsion is carried out and the variations of the strains are shown in Fig. 2 (c). The increase of the difference in these strains is not observed. Fig. 3 shows the deviation deformation, defined by subtracting the uniform deformation under the constant axial compressive force with no imperfection, at ψ = 0.0022, for cases (i) and (ii). It will be shown in the next section that these two different deformations causing the above difference in the strain distribution between cases (i) and (ii) agree with the multiple branching modes by the previous theoretical prediction [5].

BRANCHING ANALYSIS

The authors [5] developed a new theory for predicting the occurrence of strain nonuniformity without tracing entire hysteretic behaviors, for this problem. In this section the previous theory is summarized and the above numerical results are compared with the prediction by this theory. Consider the *idealized cyclic loading* (ICL) program introduced by [4], in which at each level of ψ the torsional cycles are repeated as many times as is necessary to attain a steady state and ψ is continuously increased by small increments. A continuous sequence of steady states generated under the ICL program is regarded as a continuous path in a *steady-state space*, each point in which represents a steady state. This path is referred to as a *steady-state path* (SSP). In this theory, the critical steady state at which strain nonuniformity occurs is found as a bifurcation point of an SSP. The direct formulation only in terms of state variables belonging to a steady state is carried out without tracing any convergent behavior to the steady state. Deriving rate forms of the stiffness equations with respect to ψ, we have

$$\mathbf{K}\dot{\mathbf{U}} = \dot{\mathbf{F}} \tag{1}$$

where **U** is the vector composed of unknown displacements and **F** is that derived from prescribed forced displacements. Eq. 1 has a trivial solution causing uniform deformation. The transition to a strain-nonuniform steady state is characterized by the condition that Eq. 1 has a complementary solution and this condition corresponds to the singularity of **K**. For the present cylinder model, at yielding, the rank of **K** decreases by 18 and it exhibits a multiple branching point. Some of branching modes are shown in Fig. 4. Lower-order branching mode exhibits shorter waves, analogous to *internal buckling* [1]. It can be seen that the deviation deformation causing strain nonuniformity in case (ii) shown in Fig. 3(b), agrees with the highest-order branching mode. As for the deviation deformation in case (i) shown in Fig. 3(a), no branching mode agrees with it but it can be regarded as superposition of the second one and thirteenth one.

References

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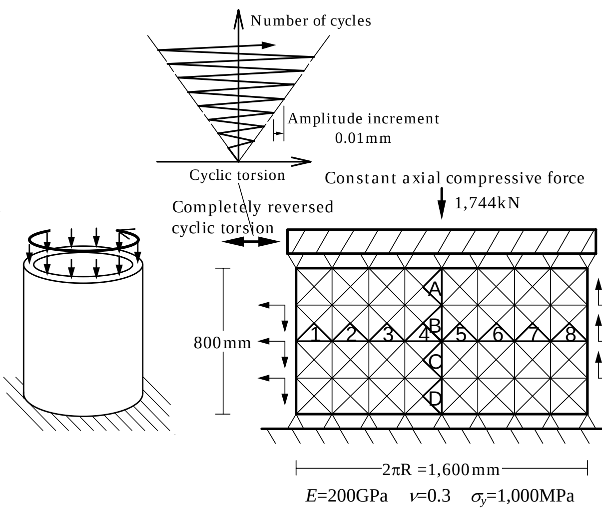

Fig. 1. Analytical model and loading conditions.

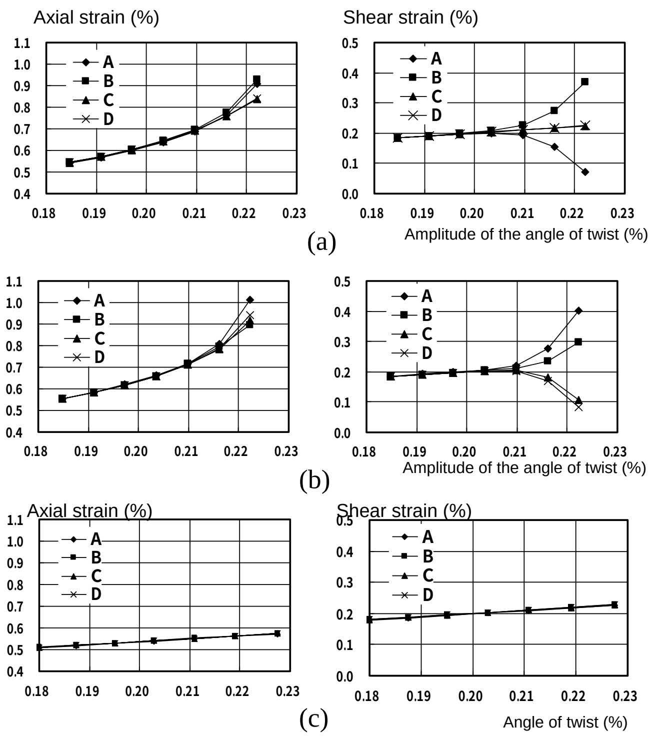

Fig. 2. Variation of the strains of axially adjacent four elements. (a) case 1. (b) case 2. (c) monotonic torsion.

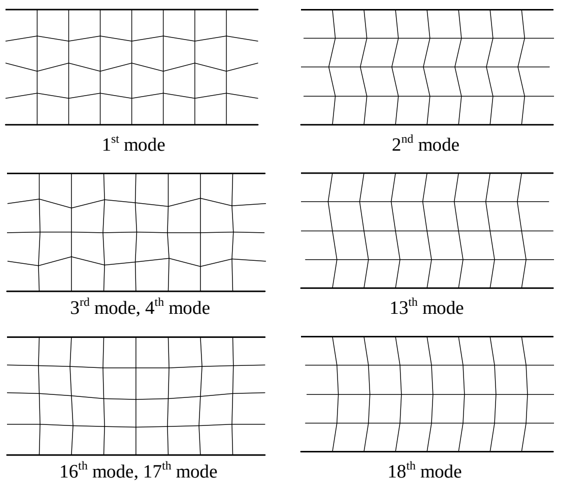

Fig. 4. Branching modes of the steady-state path.

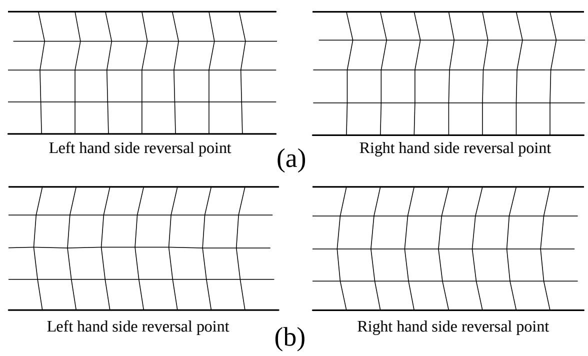

Fig. 3. Deviation deformation at ψ=0.0022.(a) case 1. (b) case 2.