SURFACE EFFECTS ON THE WRINKLES IN A STIFF THIN FILM BONDED TO A COMPLIANT SUBSTRATE

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Summary Surface effects are important to predict the mechanical behavior of nanostructures. The wrinkling of a stiff thin film bonded to a compliant substrate is studied using an energy method accounting for surface elasticity and residual surface tension. The wavelength, critical buckling strain and amplitude are obtained analytically. These results provide valuable guide to the precise design and control of the wrinkling profile in many applications ranging from stretchable electronics to precision metrology.

1. Introduction

The wrinkling of a thin film bonded to a compliant substrate has important applications such as stretchable electronics, pattern formation for micro/nano-fabrication and high precision micro and nano-metrology methods. A typical approach to fabricate the wrinkles is to deposit or transfer printing a stiff thin film on a prestrained compliant substrate. Releasing the prestrain leads to a compressive strain in the film and once this strain reaches the critical wrinkling strain, the film buckles to form wrinkles. Several models, which are based on the classic continuum theory and neglect surface effects, have been developed to explain the mechanics of wrinkling. These models are is accurate for wrinkling systems with film thickness on microscale. However, with the reduction of film thickness to nanoscale as shown in [1-3], the surface effects cannot be ignored due to the large ratio of surface area to volume. In these cases, a model accounting for surface effect is critically needed to accurately predict the wrinkling behavior. The objective of this work is to develop an energy model to study the wrinkles of thin films on a compliant substrate accounting for the surface effects including surface elasticity and residual surface tension.

2. Modeling and results

For a stiff thin film with thickness h_f, Young's modulus E_f, and Poisson's ratio ν_f on a prestrained compliant substrate with prestrain ε_{pre}, modulus E_s, Young's modulus E_s, and Poisson's ratio ν_s, the wrinkles form when the prestrain is released as shown in Fig. 1. The surface-layered-based model [4,5] considering surface elasticity through surface constitutive equations and residual surface stress through the Laplace-Young equation is adopted to study the surface effects on the wrinkles. The film is modeled as an elastic nonlinear von Karman beam with effective flexural and tensile rigidities $(EI)^* = \bar{E}_f h_f^3/12 + \bar{E}_f^s h_f^2/2$ and $(EA)^* = \bar{E}_f h_f + 2\bar{E}_f^s$ subjected to a distributed transverse loading $p(x_1) = 2\tau^0 w''$, where $\bar{E}_f^s$ is the surface elastic modulus, τ^0 is the residual surface tension, $\bar{E}_f$ is the plane strain modulus of bulk, and κ is the principal curvature of the surface, which can be approximated by the second derivative of the deflection $w(x_1)$.

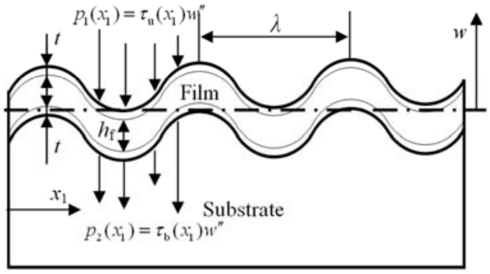

FIGURE 1. Schematic diagram of wrinkles of a stiff thin film with wavelength λ bonded to a compliant substrate.

The out-of-plane displacement of the wrinkled thin film can be represented by $w = A\cos(kx_1)$ where A is the amplitude and k is the wave number, which are to be determined by energy minimization. The total energy consists of the bending energy U_b of the beam due to wrinkling, the membrane energy U_m in the beam, the residual tension energy U_r due to the residual surface tension, and the substrate energy U_s. The total energy can be obtained by $U_{total} = U_s + U_m + U_b + U_r$. Minimization of the total energy (i.e., $\partial U_{total}/\partial A = \partial U_{total}/\partial k = 0$) gives the wave number k and amplitude A as

$$\frac{\lambda}{\lambda_0} = \sqrt[3]{1+6a_1}, \quad \frac{\varepsilon_c}{\varepsilon_c^0} = \frac{\sqrt[3]{1+6a_1} + 8a_2}{1+2a_1}, \quad \frac{A}{A_0} = \frac{\sqrt{\frac{\varepsilon_{pre}}{\varepsilon_c^0} - \frac{\varepsilon_c}{\varepsilon_c^0}\sqrt[3]{1+6a_1}}}{\sqrt{\frac{\varepsilon_{pre}}{\varepsilon_c^0} - 1}}, \tag{1}$$

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where $\lambda_0 = 2\pi h_{\mathrm{f}} \left(3\bar{E}_{\mathrm{s}} / \bar{E}_{\mathrm{f}}\right)^{-1/3}$, $A_0 = 2h_{\mathrm{f}} \sqrt{\varepsilon_{pre} - \varepsilon_c^0} (3\bar{E}_{\mathrm{s}} / \bar{E}_{\mathrm{f}})^{-1/3}$ and $\varepsilon_{\mathrm{c}}^0 = (3\bar{E}_{\mathrm{s}} / \bar{E}_{\mathrm{f}})^{2/3} / 4$ are results without surface effects, and $a_1 = \bar{E}_{\mathrm{f}}^{\mathrm{s}} \big/ \bar{E}_{\mathrm{f}} h_{\mathrm{f}}$ and $a_2 = \tau^0 / \left[h_{\mathrm{f}} (\bar{E}_{\mathrm{f}})^{1/3} (3\bar{E}_s)^{2/3} \right]$ are the two non-dimensional parameters, which characterize the effect of surface elasticity and residual surface tension, respectively. It should be noticed that the wavelength only depends on surface elasticity but not the residual surface tension. While the critical wrinkling strain and amplitude depends on both surface elasticity and residual surface tension.

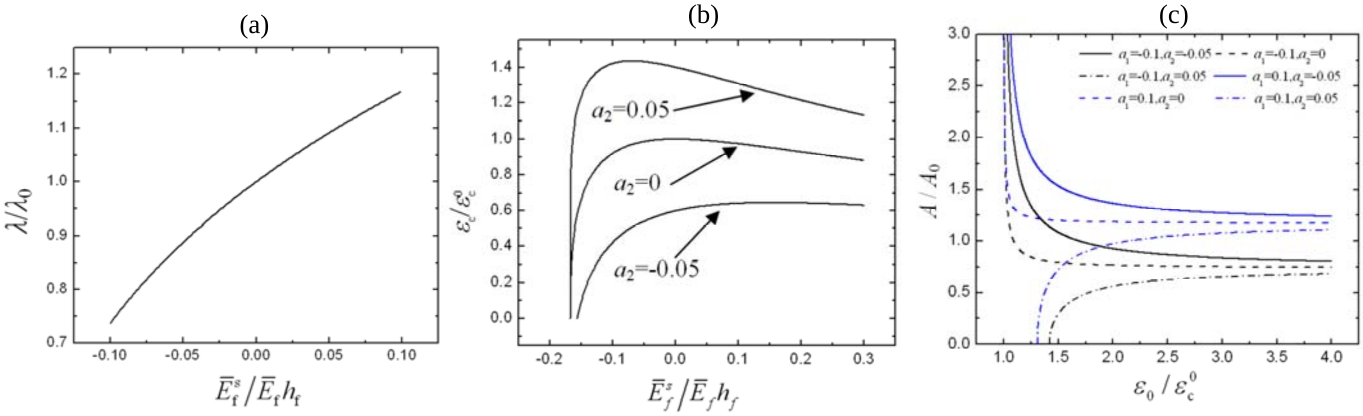

FIGURE 2. (a) The normalized wavelength versus the normalized surface elasticity, (b) The normalized critical wrinkling strain versus the normalized surface elasticity, and (c) The normalized amplitude of the wrinkles versus the normalized prestrain with different normalized surface elasticity and residual surface tension. (Reprinted with permission from Ref. 6)

Figure 2(a) shows the normalized wavelength versus the non-dimensional parameter a_1. With the increase of the surface elasticity, the normalized wavelength increases gradually. For example, when a_1 increases from -0.1 to 0.1, the normalized wavelength increases from 0.74 to 1.17.

The surface effect on the normalized critical strain is shown in Fig. 2(b). Both surface elasticity (a_1) and residual surface tension (a_2) have significant influences on the critical strain for the wrinkles. For the surface elasticity, with the increase of a_1, the critical strain increases at the beginning and decreases after a_1 passes a critical value where the critical strain reaches maximum. The critical a_1 can be easily found by taking derivative of $\varepsilon_{\mathrm{c}} \big/ \varepsilon_{\mathrm{c}}^0$ with respective to a_1 in Eq. (1) and letting it to be zero. The maximum critical wrinkling strain can then be obtained by substituting the critical a_1 back to $\varepsilon_{\mathrm{c}} \big/ \varepsilon_{\mathrm{c}}^0$. The lengthy analytical expressions are functions of a_2 and are not given here due to page limit. For the residual surface tension, with the increase of a_2, the critical strain increases. For example, at $a_1 = 0$, the normalized critical strain increases from 0.6 to 1.4 as a_2 increases from -0.05 to 0.05.

The normalized amplitude by the counterpart without surface effect, $A \big/ A_0$, versus the normalized prestrain $\varepsilon_{pre} / \varepsilon_{\mathrm{c}}^0$ is shown in Figure 2(c). The black curves show the results at $a_1 = -0.1$ with different a_2 values -0.05, 0 and 0.05 and the blue curves are $a_1 = 0.1$. These curves show that a_1 and a_2 have significant influences on the normalized amplitude. As a_1 increases, the normalized amplitude increases while as a_2 increases, the normalized amplitude decreases. It should be noticed that the effect of a_2 decreases as the prestrain increases. This is can be easily explained by $A \big/ A_0$ in Eq. (1). When $\varepsilon_{pre} >> \varepsilon_{\mathrm{c}}^0$, $A \big/ A_0 \approx \sqrt[3]{1+6a_1}$, which is independent of a_2.

4. Conclusions

In conclusion, an energy model is developed to investigate the surface effects on the wrinkles of a thin stiff film bonded to a compliant substrate. The wavelength, amplitude and critical wrinkling strain are obtained analytically and the results might be helpful for the design of the film/substrate system for the stretchable nano-electronic devices.

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