BUCKLING OF POROUS ELASTIC SANDWICH PLATES

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Summary In the framework of a general stability theory for three-dimensional bodies, the buckling analysis has been carried out for two common types of sandwich plates: a circular plate subject to radial compression and rectangular plate subject to biaxial compression. The size effect and influence of coating properties on the stability of elastic sandwich plates with a porous core have been analyzed.

EXTENDED SUMMARY

The problem of equilibrium stability for deformable bodies is of major importance both from theoretical and practical points of view, because the exhaustion of load-carrying capability and collapse of buildings and engineering structures quite often occurs due to buckling under external loads. In the case of elastic medium, the stability theory is extensively developed for classic non-polar materials. There is large number of studies on stability both for thin and thin-walled bodies in the form of rods, plates and shells, and for massive (three-dimensional) bodies. However, due to the increasing number of new structural materials, the problem of stability analysis for bodies with a microstructure becomes relevant. The example of such materials are porous materials. Engineering constructions made of porous materials, especially metal and polymer foams, have different applications in the last decades [2, 3, 5, 7]. The reason for this is the advantages of such materials: better density-stiffness ratios in comparison with classical structural materials, the possibility to absorb energy, etc. As a rule, these constructions have a composite structure (porous core covered by hard and stiff shell), which is necessary for corrosion or heat protection, and optimization of mechanical properties in the process of loading.

Problem statement and methods of solution

The foams are cellular structures consisting of a solid metal (for example aluminium, steel, copper, etc.), or polymer (polyurethane, polyisocyanurate, polystyrene, etc.) and containing a large volume fraction of gas-filled pores. There are two types of foams. One is the closed-cell foam, while the second one is the open-cell foam. The defining characteristic of metal and polymer foams are the very high porosity: typically, well over 80%, 90% and even 98% of the volume consists of void spaces. From the mechanical point of view, foam is a very complex material, and it seems appropriate to describe it using a model of a micropolar medium or Cosserat continuum [1, 4, 6, 8, 10, 15]. For this model, the kinematics of a body is more complex then in the classical non-polar continuum: each material particle of the micropolar medium has an additional inner degree of freedom – a microrotation. Because of this, a deformation in the Cosserat theory of elasticity is described not by one, but by two strain measures [11, 12] – the stretch tensor and wryness tensor. In general case, these tensors are nonsymmetrical, which considerably complicate the solution of stability problems for micropolar media. This approach allows, in particular, modeling the size effect, which is typical for foams and consists in dependence of material behavior on the ratio between the foam cell size and sample size.

In the present research, the stability of porous elastic sandwich plates have been studied in the framework of a general stability theory for three-dimensional bodies. We have carried out the buckling analysis for two common types of plates: a circular plate subject to radial compression and rectangular plate subject to biaxial compression. For the modeling of sandwich plates with a porous core, a special approach is used [13, 14]: the behavior of the porous core has been studied in the framework of the Cosserat continuum, while the classical (non-polar) continuum model is used to describe the properties of the upper and lower coatings. The semi-inverse method of nonlinear elasticity is used to determine the subcritical stress-strain state of sandwich plates under finite strains. In the case of physically linear model of isotropic material [9], the initial state of equilibrium for the considered problems has been found analytically. Using the linearization method in a vicinity of an initial state [16], the neutral equilibrium equations are derived, which describe the perturbed state of sandwich plates. For a few commonly used materials these equations are solved numerically [13, 14]. The spectra of the critical values of radial compression (for the circular plate) and the critical curves in the plane of loading parameters (for the rectangular plate) have been found, as well as the corresponding buckling modes. The stability regions have been constructed.

Main results

We have studied in detail the influence of elastic properties as well as plate geometry on the loss of stability. Special attention has been given to the analysis of how the coatings thickness and rigidity affects the stability of sandwich plates with a porous core. It has been found that a decrease in the size of a plate increases its stability, and for plates with a less rigid coatings the size effect on the stability would be more substantial than for plates with a more rigid one. Such behavior is due to the influence of couple stresses and confirms the need to take the microstructure into account when analyzing the stability of bodies and engineering structures made of highly porous materials if objects of study are relatively small. At the same time, according to the results, the influence of the microstructure of porous material on the stability of large sandwich plates and sandwich plates with thick rigid coatings is very small and, if necessary, it can be neglected.

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It should be noted that the qualitative results, obtained in this research for highly porous materials, can be generalized to larger class of materials with complex microstructure, for describing the behavior of which the model of a micropolar medium or Cosserat continuum is used.

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