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Summary This paper reviews the theoretical basis for the dynamic numerical analysis to examine the elastic stability of the multi-folding microstructure(MFM) [1]. The analysis allows for geometrical non-linearity with localization based upon bifurcation theory [2]. We have obtained interesting bifurcation paths from the hill-top singular point on a foldable truss. The authors suggest that the understanding of snapthrough behaviour for folding mechanics with imperfection will be very interesting in different situation from the perfect equilibrium paths.

NONLINEAR FOLDING ANALYSIS FOR A MULTI-FOLDABLE TRUSS

In this section, we consider the folding mechanism for the pantographic folding structures subject to a vertical load at the top of the multi-folding system. Let us consider the analytically ideal model of the multi-layered trusses. The system is a pin-jointed elastic truss, and all nodes of the system displace vertically only. No allowance is made the friction, gravity or buckling for each member by itself in this paper.

Theoretical approach for the MFM model

We consider that a structural system contains several pin-jointed diamond trusses with left-right symmetry. In general, it is more complex to make the formula of the equilibrium equations with geometrical condition. It looks that solving the equations are more difficult. In particular, if we used the engineering strain for the total strain energy, it would be so far from theoretical work with mathematical proof. Hence, for example, even though the system has beautiful symmetry, it has never been so useful to know the reason why it happens on symmetry-breaking issue. In case of this model, we have to consider the theoretical bifurcation analysis including with nonlinear dynamics.

Now, let us consider a theoretical estimation for the multi-folding truss model. We assume a periodic height for each layer of $h_i = \gamma_i L$ where the width L of the truss is fixed. Therefore, an initial length for each bar in the geometry of the figure is expressed as $\ell_i = \sqrt{L^2 + h_i^2} = L\sqrt{1 + \gamma_i^2}$, ($i = 1, \dots, n$). The deformed length of each bar denoted as $\hat{\ell}_i$, is a function of the height and the nodal displacement variables

$$\hat{\ell}_i = L\sqrt{1 + (\gamma_i - q_i(\bar{v}_i, \bar{v}_{i+1}))^2}, \quad i = 1, \dots, n \quad (1)$$

where $\gamma_i = h_i/L > 0$, $q_i(\bar{v}_i, \bar{v}_{i+1}) = \bar{v}_i - \bar{v}_{i+1}$, $q_n(\bar{v}_n, \bar{v}_{n+1}) = \bar{v}_n$, $\bar{v}_i = v_i/L$, ($i = 1, \dots, n$).

Using Green's expression for strain we apply the strain in each elastic bar as

$$\varepsilon_i \equiv \frac{1}{2} \left\{ \left(\frac{\hat{\ell}_i}{\ell_i} \right)^2 - 1 \right\} = \frac{1}{2} \left\{ \frac{1 + (\gamma_i - q_i)^2}{1 + \gamma_i^2} - 1 \right\}, \quad (2)$$

The total energy of a half of the system is given by

$$\mathcal{V} = \sum_{i=1}^n \frac{EA_i \ell_i}{2} (\varepsilon_i)^2 - f^* \bar{v}_1 L = \sum_{i=1}^n \frac{EA_i L \sqrt{1 + \gamma_i^2}}{2} \frac{1}{4} \left\{ \frac{1 + (\gamma_i - q_i)^2}{1 + \gamma_i^2} - 1 \right\}^2 - \frac{f}{2} \bar{v}_1 L \quad (3)$$

in where, f^* is a half of load $f^* = f/2$, and $\bar{v}_{n+1} = 0$. In the case of the same height for each layer, $\gamma = \gamma_i$ and the same stiffness for each member, $EA = EA_i$. Then, the total potential energy can be described as

$$\mathcal{V} = \frac{\beta L}{8} \sum_{i=1}^n (q_i)^2 (q_i - 2\gamma)^2 - \frac{f}{2} \bar{v}_1 L \quad (4)$$

where the stiffness parameter $\beta = EA/(1 + \gamma^2)^{3/2}$, which is a function of γ . From Eq. (4), we can obtain the equilibrium equations based on the principal of minimum energy[2].

Nonlinear equilibrium equations

By allowing for symmetric model, we can therefore consider nonlinear equilibrium equations based on the total strain energy theoretically.

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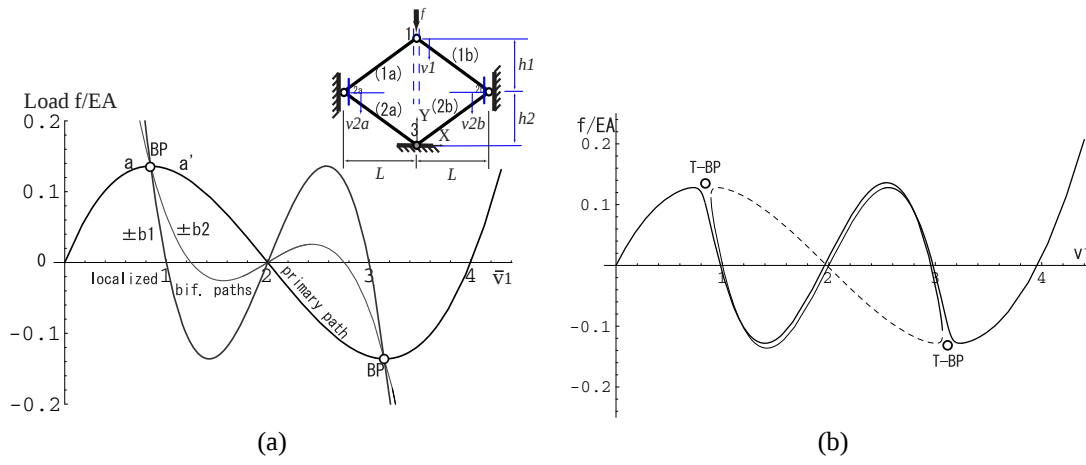


Figure 1. Mathematical results for a foldable truss without any contact ($h_1 = h_2 = 1, L = 1, n = 2$); (a) Equilibrium paths for perfect diamond truss, (b) Breaking bifurcation paths for imperfect model ($\epsilon = 2\%$ less of h_1).

For the 1st and the second equilibrium equations chained

$$F_1 = \frac{\partial \mathcal{V}}{\partial v_1} = \frac{\partial \mathcal{V}}{\partial q_1} \frac{\partial q_1}{\partial v_1} \frac{1}{L} = \frac{\beta}{2} q_1 (q_1 - \gamma) (q_1 - 2\gamma) - \frac{f}{2} = 0$$

$$f = \beta (\bar{v}_1 - \bar{v}_2) ((\bar{v}_1 - \bar{v}_2) - \gamma) ((\bar{v}_1 - \bar{v}_2) - 2\gamma), \quad (5)$$

$$F_2 = \frac{\partial \mathcal{V}}{\partial v_2} = \sum_{j=1}^2 \frac{\partial \mathcal{V}}{\partial q_j} \frac{\partial q_j}{\partial v_2} \frac{1}{L} = \beta \sum_{j=1}^2 q_j (q_j - \gamma) (q_j - 2\gamma)$$

$$= \beta \{ (\bar{v}_1 - \bar{v}_2) ((\bar{v}_1 - \bar{v}_2) - \gamma) ((\bar{v}_1 - \bar{v}_2) - 2\gamma) + \bar{v}_2 (\bar{v}_2 - \gamma) (\bar{v}_2 - 2\gamma) \} = 0 \quad (6)$$

For example, let us consider $n = 2$ as a diamond truss. These equations equal 0 in the limit value problem of total potential energy. It is possible to solve all variables $\bar{v}_i$ by substituting the obtained solutions into the next limited condition using the theorem of implicit function.

$$F_2(\bar{v}_1, \bar{v}_2) = 0 \rightarrow \bar{v}_2 = \mathcal{F}_2(\bar{v}_1); \quad F_1(\bar{v}_1, \bar{v}_2) = F_1(\bar{v}_1, \mathcal{F}_2(\bar{v}_1)) = 0, \quad (7)$$

where $\mathcal{F}(\cdot)$ denotes a function of the nonlinear solutions. Finally, we obtain all solutions completely as perfect equilibrium paths. If the MFM system has a imperfection ϵ such as $\gamma_1(\epsilon) = \gamma + \epsilon$, you can show the different situation from the perfect equilibrium paths with the hill-top bifurcation point for (a) the perfect model and for (b) the imperfection model, respectively, in **Figure 1**.

REMARKS

This paper clearly shows that there are different bifurcation paths and a primary nonlinear equilibrium path, based on a theoretical solution approach. And there is a hill-top bifurcation point or there are multiple bifurcation equilibrium paths through two BPs. This nonlinear phenomena appears to be folding behaviour that limits the holding supports in a real structure like an umbrella. In future, we will show the numerical results as attractors for nonlinear strange behaviour.

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