LOCALIZATION AND LOSS OF MACROSCOPIC ELLIPTICITY IN MICROSTRUCTURED SOLIDS

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Summary Loss of ellipticity in a continuum is associated with localization of deformation patterns, because it allows for macroscopically discontinuous equilibrium solutions in nonlinear solids. However, a localized deformation pattern does not necessarily occur when a solid loses ellipticity "on the average" i.e. in its macroscopic (homogenized) response. The key to understanding the connection between a macroscopic loss of ellipticity and a localization of the solid's deformation pattern lies in the post-bifurcation behavior of the solid under investigation. The connection is elucidated for the case of media with periodic microstructures through an investigation of the post-bifurcation behavior of finitely strained, nonlinear, layered solids.

INTRODUCTION

Loss of ellipticity in a continuum is associated with localization of deformation patterns [1], because it allows for macroscopically discontinuous equilibrium solutions in nonlinear solids. Investigations in various types of microstructured solids have also shown that a loss of ellipticity in their macroscopic (homogenized) properties leads to equilibrium solutions with highly localized deformation fields, as in the case of kinkbands in axially compressed, fiber-reinforced composites or cellular-type structures (honeycomb). Consequently a macroscopic loss of ellipticity is widely considered as a precursor of localization of deformation in solids.

However, a localized deformation pattern does not necessarily occur when a solid loses ellipticity "on the average" i.e. in its macroscopic (homogenized) response. The key to understanding the connection between a macroscopic loss of ellipticity and a localization of the solid's deformation pattern lies in the post-bifurcation behavior of the solid under investigation. The connection is elucidated for the case of media with periodic microstructures through an investigation of the post-bifurcation behavior of finitely strained, nonlinear, layered solids. It is shown that depending on constitutive properties and microgeometry, solids that lose macroscopic ellipticity at certain loads can sustain higher macroscopic loads, thus avoiding localization of deformation. An asymptotic post-bifurcation analysis for these composites investigates under what conditions a loss of macroscopic ellipticity implies or avoids localization of deformation and gives examples for each case.

RELATION BETWEEN MICROBUCLING, LOSS OF ELLIPTICITY AND LOCALIZATION

Failure in composite materials is a fundamental as well as an extremely diverse issue in solid mechanics. Questions about what constitutes failure, when does failure start and whether it is possible to predict the onset of failure by investigating the effective (homogenized) properties of the solid, are problems of fundamental interest for all composites. In the interest of (relative) simplicity, attention is hereby restricted to periodic, finitely strained non-linear solids. A rigorous connection has been established by [2] between bifurcation instability at the microscopic level and loss of rank-one convexity of the homogenized moduli in finitely strained periodic elastomers of infinite extent. More specifically, it was shown that if the wavelength of the bifurcation eigenmode is infinite (compared to the unit cell size), the corresponding instability of the periodic principal solution can be detected as a loss of ellipticity ($Det[\mathbf{n.L.n}] = 0$) of the corresponding one cell homogenized tangent moduli **L** of the solid (see **Figure 1**a).

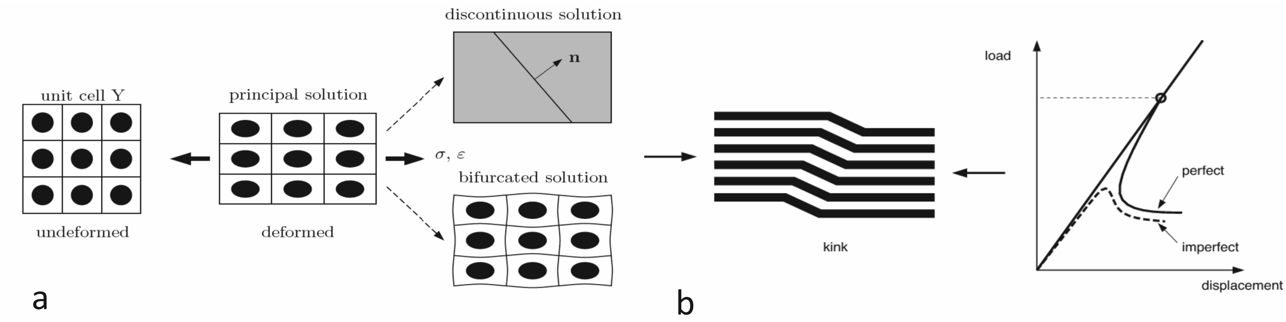

Figure 1: a) *Relation between microscopic bifurcation of a periodic solid and loss of ellipticity in its macroscopic moduli. b) If post-bifurcated solution is unstable (as in the example of the axially compressed composite), a localized deformation pattern (kink-band) evolves.*

It is frequently observed (as in the case in cellular solids and fiber-reinforced composites), that for loads past the critical one corresponding to the onset of the bifurcation in the perfect solid, that the structure evolves into a highly localized deformed pattern, thus leading to the association of a loss of ellipticity in the macroscopic (homogenized) moduli with the localization of a deformation pattern (see **Figure 1**b**).** However, situations exist where the onset of a long wavelength bifurcation instability does not lead to a localized deformation pattern, in spite of the fact that the homogenized solid has lost its macroscopic ellipticity, and this is the subject of the present work.

STABLE POST-BUCKLING OF A LAYERED COMPOSITE

To provide an example of a solid that loses the ellipticity of its homogenized moduli at finite strain levels, but which does not exhibit localized deformation patterns at loads near the critical one, we solve the problem of an axially compressed, layered, hyperelastic composite. As seen in **Figure 2**a**,** the bifurcation mechanism corresponding to the macroscopic loss of ellipticity corresponds to a long wavelength eigenmode. The post-bifurcation solution can be solved analytically due to the state of constant strain in each layer, as also seen in **Figure 2**a below.

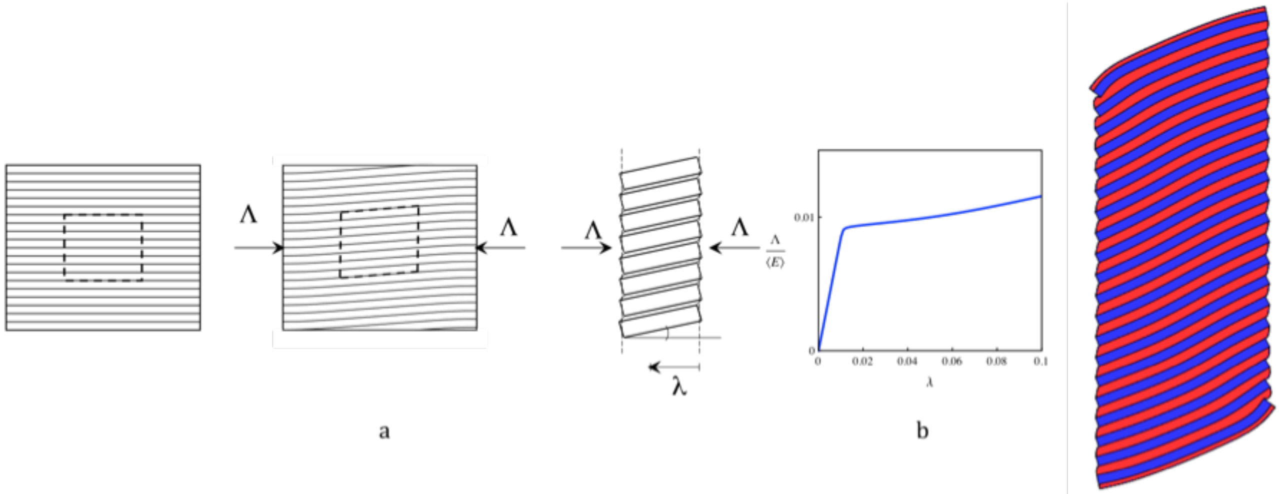

Figure 2: a) *Post-bifurcation mechanism for finitely strained, axially compressed layered solid. b) Stable post-bifurcation behavior of finite-sized, axially compressed layered specimen (notice absence of kink-band).*

In addition to the exact post-bifurcation solution for the perfect layered solid, detailed numerical calculations of a finite, axially compressed structure with a small initial imperfection show that the overall force displacement is monotonically increasing, thus precluding localized deformation patterns near the critical load, as seen in **Figure 2**b. Some additional examples using two-dimensional periodic, particle-reinforced composites are also presented.

CONCLUSIONS

The above work shows, through the analytical solution of a microstructured, hyperelastic solid and also through related numerical calculations, that a finitely strained, rate-independent solid can loose the ellipticity of its homogenized moduli at a certain critical load, but this does not necessarily imply the appearance of localized deformation patterns, in contrast to currently held opinion in the engineering literature.

References

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