STAMPING AND WRINKLING OF ELASTIC PLATES

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Summary When a thin flat sheet is compressed inside a spherical mold, the resulting orthoradial compressive stresses induce the formation of wrinkles. The amplitude of these wrinkles is geometrically constrained by the opposite walls of the mold. We first show through a 1D analytical model how the wavelength of the wrinkles is driven by this constraint and reaches a finite value as the gap vanishes. This simplified approach is extended to describe the cascade of wrinkles observed in the original axisymmetric situation of the spherical mold.

Mechanical instabilities are observed when elastoplastic plates are stamped into biaxially curved dies [1,2] (Fig. 1a). As the mold is progressively closed, orthoradial compression rises near the edges of the plate which induces the formation of radial wrinkles (Fig. 1b,c). Here we study circular elastic plates stamped into spherical dies under high compressions where the gap separating the opposite molds reaches the order of the plate's thickness.

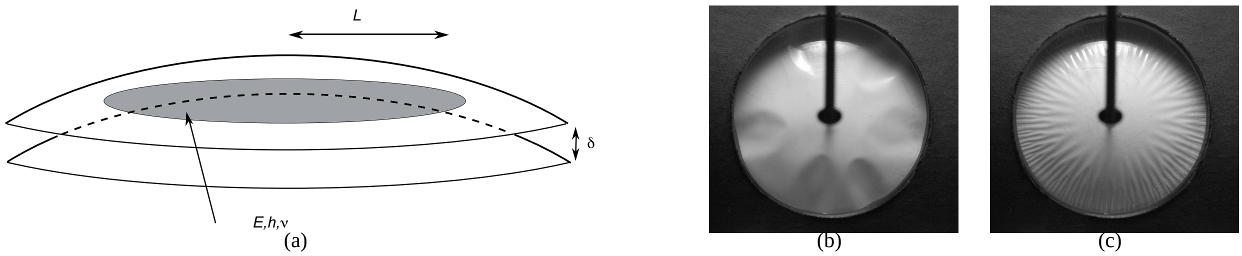

Figure 1. (a): 2D Experimental setup. A circular plate of radius L and thickness h is compressed between two rigid hemispherical dies of radius R. The gap between the spheres is noted δ. (b, c): Top view. Experimental observations of wrinkles when δ is decreased.

As can be seen on fig. 1b,c, the number of wrinkles increases as $\delta \to h$, going from isolated structures to a smooth wrinkled pattern. However, the number of wrinkles reaches a finite value as the gap is closed. Moreover, the central part of the plate remains unwrinkled. In order to undertand these two characteristics, and especially the former, we first describe the buckling of a one-dimensional plate constrained in a spacer.

We consider an elastic plate (Young's modulus E and Poisson's ratio ν) of length L, thickness h and unit width. In-plane displacements are imposed at the end points $u(\pm L/2) = \pm\Delta/2$ ((Fig. 2a). The plate is constrained by a spacer delimiting a gap of size δ. We note A the wrinkles amplitude, *i.e.*, the amplitude of the median plane, with $A = \delta - h$. In the small slope approximation, the deflection of a single wrinkle is taken as $w(-\lambda/2 \leq x \leq \lambda/2) = A/2[1 + \cos(2\pi x/\lambda)]$, where λ is the wavelength. The associated bending energy is thus $\mathcal{E}_b = \frac{B}{2}\int_{-\lambda/2}^{\lambda/2} w_{,xx}^2$, with $B = Eh^3/[12(1-\nu^2)]$ the bending modulus of the plate [4]. According to the in-plane equilibrium equation $\partial_x \sigma = 0$, the in-plane stress σ and strain ϵ are constant in the wrinkle, leading to $\epsilon = [\int_{-\lambda/2}^{\lambda/2}(u_{,x} + w_{,x}^2/2)]/\lambda$. The stretching energy can be written as $\mathcal{E}_s = \frac{C}{2}\epsilon^2\lambda$, where $C = Eh/(1-\nu^2)$ is the stretching modulus of the plate. Here we assume the existence of n similar wrinkles. Imposing $n\lambda = L$ and $n\Delta_{one} = \Delta$ in the expression of the bending and stretching energies, and minimizing the total energy with respect to the number of wrinkles leads:

$$n^2 = \frac{12\,(\Delta/L)\,L^2}{3\,\pi^2\,A^2 + 8\,\pi^2\,h^2} \tag{1}$$

We note that, for $A \to 0$, the number of wrinkles tends to a finite value $n_{max} = 0.39(L/h)\sqrt{\Delta/L}$. In fig. 2b, we plot the evolution of the number of blisters as a function of the dimensionless amplitude A/h. The "extensible" curve corresponds to eq. 1. The "inextensible" curveone corresponds to the classical approximation of the deformation of elastic lines which consists in setting the stretching energy to zero. This assumption cannot be applied if the amplitude of the plate is constrained to be of order of the thickness, as stretching becomes unavoidable.

To validate eq. 1 we measured experimentally the maximum number of wrinkles as a function of the non dimensional number $(L/h)\sqrt{\Delta/L}$ (Fig. 3a). The dependence is indeed linear but the prefactor is about half of the predicted value. A plausible explanation of this difference is the presence of flat parts along the plate. Eq. 1 thus overestimates the number of wrinkles.

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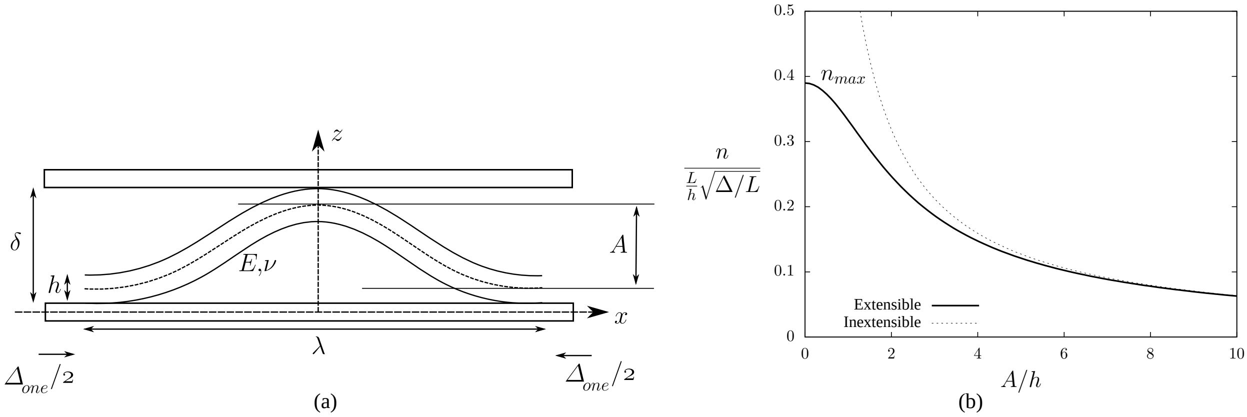
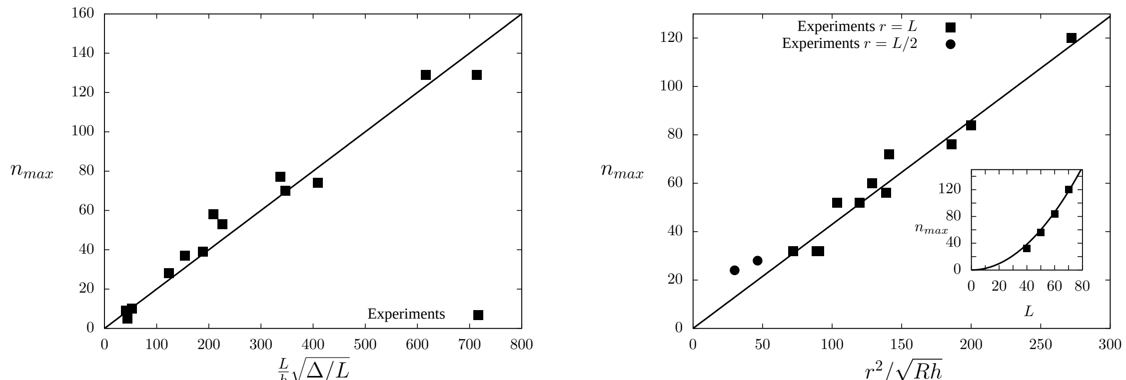


(a) (b)

Figure 2. (a) 1D Configuration: a plate of length L, thickness h and unit width is constrained to both endpoints displacements $u(\pm L/2) = \pm\Delta/2$ and prescribed maximal deflection δ. We consider that n similar wrinkles are formed ($\Delta_{one} = \Delta/n$) of wavelength $\lambda = L/n$ and amplitude A, with $A = \delta - h$. (b) Evolution of the number of wrinkles as a function of the dimensionless amplitude. Comparaison between the results of the extensible and inextensible models

Finally, we come back to our initial axisymmetric problem of stamping of a circular elastic plate in a spherical mold. The one dimensional scaling law for the maximal number of wrinkles $n_{max} \sim (L/h)\sqrt{\Delta/L}$ can be adapted to this situation by noting that the typical strain in a plate forced to match a spherical shape scales as $\Delta/L \sim r^2/R^2$ [5], leading to $n_{max} \sim r^2/Rh$, where r is the radial distance from the center of the plate. The experimental results shown in fig. 3b agree well with the prediction for the maximal number of wrinkles. The flat part in the middle can be explained by looking at the detailed stress field on the curved plate [5]: since stresses are tensile up to a distance close to one half of the plate radius, wrinkles are not expected to appear. We have shown on a one dimensional example that taking into account



(a) (b)

Figure 3. (a): 1D Experiments: Maximal number of wrinkles for polypropylene films ($E = 2500$MPa, $\nu = 0.4$, Innovia Films) of length 250mm, width 25mm and thicknesses ranging from 15μm to 250μm, with $\Delta/L = 0.2\%$ and 0.3%. The solid line corresponds to the best fit of the data: $n = 0.20(L/h)\sqrt{\Delta/L}$. (b): 2D Experiments: Maximal number of wrinkles as a function of r^2/Rh. The solid line corresponds to the best fit of the experimental data $n = 0.43r^2/Rh$. Insert: n as a function L showing the quadratic dependance.

in-plane stretching is essential to accurately describe the behavior of an elastic plate which is both constrained axially and with a prescribed maximal deflection. This observation results in a maximum number of wrinkles as one decreases the amplitude with fixed endpoints displacements, which is confirmed in one- and two-dimensional experiments.

References

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