NUMERICAL MODEL FOR NONLINEAR STABILITY ANALYSIS OF SPATIAL FRAMES WITH SEMI-RIGID CONNECTIONS

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Summary This paper presents a beam element for the numerical analysis of nonlinear stability problems encountered at semi-rigid frames composed of the straight and prismatic thin-walled beam members. The equations of equilibrium are derived using the updated Lagrangian incremental description and the nonlinear displacement field of asymmetric thin-walled cross-sections. To account for the semi-rigid connection behaviour, a special transformation procedure is developed. The numerical algorithm is implemented in a computer program and its reliability is validated trough the test problems.

INTRODUCTION

Stability limit state evaluations of frame-type structures are usually carried out under the assumption that structural connections are either perfectly rigid or perfectly hinged [1]. Such an approach simplifies the stability analyses significantly, but often fails to represent the real structural behaviour, because the connections of actual structures exhibit a flexible behaviour falling in between the two idealised cases, filling the entire flexibility spectrum from hinge-like connections, over semi-rigid connections, to rigid connections [2]. This work presents a large-displacement beam model for the nonlinear stability analysis of frames composed of the straight and prismatic beam members, with semi-rigid connections. Displacements are allowed to be large, but strains are assumed to stay small. External loads are static and conservative. The element geometric stiffness is derived using the updated Lagrangian (UL) description and the nonlinear displacement field of an asymmetric thin-walled cross-section, which includes the second-order displacement terms due to large rotation effects. At the end of each loading step, the updating of nodal orientations is performed using the transformation rule which applies for semitangential incremental rotations [3]. Semi-rigid connections are allowed to occur at finite element nodes by modifying the stiffness matrices and nodal vectors of a nonlinear beam element for which rigid connections at the end nodes are assumed. A special transformation matrix is derived for that purpose.

FINITE ELEMENT FORMULATION

Fig. 1 shows a conventional 14 degrees-of-freedom beam-type finite element with an asymmetric cross-section of a wall thickness t. A right-handed Cartesian coordinate system (z, x, y) is chosen in such a way that z-axis coincides with the beam axis passing through the centroid O of each cross-section, while the x- and y-axes are the principal inertial axes of the cross-section. The shear centre S of the cross-section is defined by the position co- ordinates x_s and y_s.

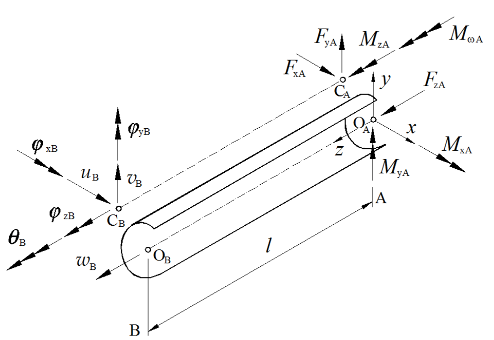

Figure 1. Thin-walled beam element: nodal displacements and nodal forces.

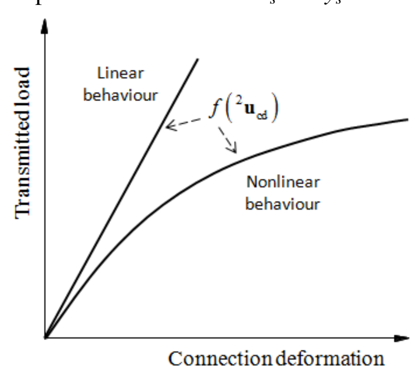

Figure 2. Semi-rigid connection behavior.

Using the UL formulation [1], the incremental equilibrium equations for the beam element from Fig.1can be written as:

$$\mathbf{f} = \mathbf{k}_{\mathrm{T}}\,\mathbf{u}, \quad \mathbf{k}_{\mathrm{T}} = \mathbf{k}_{\mathrm{E}} + \mathbf{k}_{\mathrm{G}}, \quad \mathbf{f} = {}^{2}\mathbf{f} - {}^{1}\mathbf{f} \qquad (1)$$

where $\mathbf{k}_{\mathrm{T}}$, $\mathbf{k}_{\mathrm{E}}$ and $\mathbf{k}_{\mathrm{G}}$ are, respectively, the tangent, elastic and geometric stiffness matrices, while vectors $\mathbf{u}$ and $\mathbf{f}$ contain corresponding nodal displacement and force increments, respectively. In order to include semi-rigid connection behavior into the numerical procedure, the data for load-deformation relationships of the given connection must be mathematically represented. These relations can be expressed in a general form as:

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$$^{2}\mathbf{f} = f\left({}^{2}\mathbf{u}_{\text{cd}}\right), \quad \mathbf{f} = \mathbf{S}_{\text{T}}\,\mathbf{u}_{\text{cd}}, \quad \mathbf{S}_{\text{T}} = \partial\,{}^{1}\mathbf{f}/\partial\,{}^{1}\mathbf{u}_{\text{cd}} \tag{2}$$

in which f is a known mathematical function, linear or nonlinear one, Fig. 2, ${}^{2}\mathbf{u}_{\text{cd}}$ is the connection (nodal) deformation vector of the beam element, containing the additional nodal displacements due to semi-rigidity of connections, while $\mathbf{S}_{\text{T}}$ represents the tangent connection stiffness matrix of the beam element at the beginning of the current incremental step, containing tangent stiffness parameters pertaining to each nodal d.o.f. Assuming no couplings take place, the tangent connection stiffness matrix has the diagonal form, i.e.

$$\mathbf{S}_{\text{T}} = \left\lceil \mathbf{S}_{\text{TA}} \quad \mathbf{S}_{\text{TB}} \right\rfloor, \quad \mathbf{S}_{\text{T}i} = \operatorname{diag}\left(S_{wi}, S_{ui}, S_{vi}, S_{\varphi zi}, S_{\varphi xi}, S_{\varphi yi}, S_{\theta i}\right), \quad i = \text{A, B} \tag{3}$$

Furthermore, the incremental displacement vector from Eq. (1) can be defined as a difference of the displacement vector pertaining to the semi-rigid element, $\mathbf{u}_{\text{SR}}$, and the connection incremental deformation vector from Eq. (2), i.e.

$$\mathbf{u} = \mathbf{u}_{\text{SR}} - \mathbf{u}_{\text{cd}} = \mathbf{u}_{\text{SR}} - \mathbf{S}_{\text{T}}^{-1}\,\mathbf{f} \tag{4}$$

By substituting Eq. (4) into (1), it follows:

$$\mathbf{f} = \left(\mathbf{I}_{14} + \mathbf{k}_{\text{T}}\,\mathbf{S}_{\text{T}}^{-1}\right)^{-1}\mathbf{k}_{\text{T}}\,\mathbf{u}_{\text{SR}} \tag{5}$$

where $\mathbf{I}_{14}$ denotes the 14×14 identity matrix. Eq. (5) transforms the nonlinear beam element into a form which has capabilities of performing the nonlinear stability analysis of semi-rigid frames.

EXAMPLE

Blandford's one-bay one-storey space frame with fixed bases and loaded by the vertical forces, each of intensity F, is shown in Fig. 3. All the frame members are made of W12×53 sections. The length of each beam and column is L = 144 in. The material moduli are E = 29,000 ksi and G = 11,600 ksi.

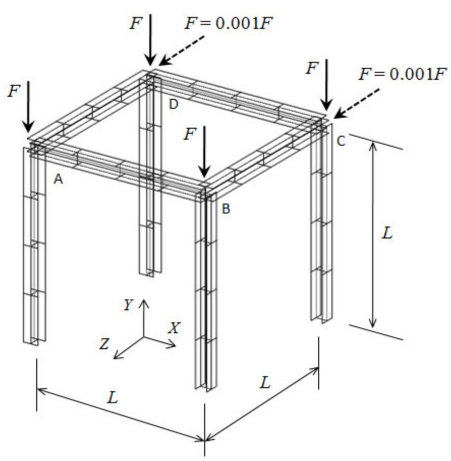

Figure 3. Blandford's space frame: configuration and load condition.

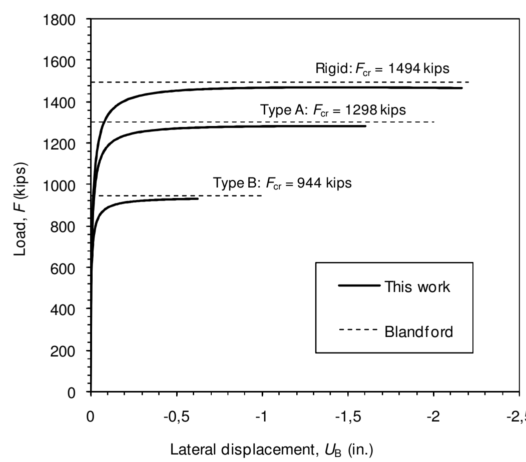

Figure 4. Blandford's space frame: elastic load-deflection curves – lateral deflection in the global X-axis direction.

The elastic sway-twist instability behaviour is analysed. To perform the nonlinear stability analysis, two perturbation forces acting in the Z-axis direction, each of intensity 0.001F, are applied at corners C and D. The analysis is performed for the rigid and two semi-rigid beam-to-column connection types: $S_{\varphi z} = 3GI_t/L$, $S_{\varphi x} = 5EI_x/L$, $S_{\varphi y} = 5EI_y/L$ and $S_\theta = 2.037EI_\omega/L$ for type A; $S_{\varphi z} = GI_t/L$, $S_{\varphi x} = EI_x/L$, $S_{\varphi y} = EI_y/L$ and $S_\theta = 0$ for type B. The instability load-deflection curves obtained by using the mesh configuration consisting of four elements per member are shown in Fig. 4, which compare very well with the results obtained by Blandford using the same FE mesh and the eigenvalue approach [4].

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