ULTRA-HIGH STIFFNESS AND DAMPING COMPOSITES AND STRUCTURES DUE TO CONSTRAINED MECHANICAL INSTABILITIES

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Summary Composites of extremely high dynamic stiffness and damping can be achieved by embedding one phase that undergoes a mechanical instability into a stiff constraining matrix phase. The combination of positive and negative (incremental) stiffness of the phases gives rise to anomalies in the overall viscoelastic performance, while the stiff matrix stabilizes the required instability in the inclusion phase. This concept applies equally to material systems and composite structures. We present a comprehensive stability analysis and demonstrate the bound-exceeding performance of such composites for both composite materials and viscoelastic structures, and we present experimental confirmation.

INTRODUCTION

Combining several materials into a composite permits the creation of new materials with overall properties tunable via the geometric composite arrangement and a careful choice of the constituent materials with favorable characteristics. The probably simplest example is a particle-matrix system, in which particulate inclusions of one material are embedded in and thus enhance the mechanical behavior of a matrix material. Composite materials theory limits the attainable overall elastic moduli of a composite in terms of the properties of its constituents, presuming positive-definiteness (positive stiffnesses) for each constituent to ensure overall stability. In particular, the classical models of Reuss/Voigt and Hashin/Shtrikman provide upper and lower bounds on the overall stiffness. Presuming materials with positive stiffness, these bounds show that the overall stiffness of the composite cannot exceed the stiffness of its stiffest constituent. Admittance of a negative-stiffness phase (i.e., one phase whose elasticities violate positive-definiteness), however, allows the composite material to strongly exceed these classical bounds [1]. Therefore, the overall performance of such composites can be dramatically altered, and stiffness and damping can be tuned to an extreme if one allows for temporarily negative elastic moduli in the inclusion phase. Such incremental negative moduli (commonly referred to as negative stiffness) entail instability; e.g. a free-standing body of negative stiffness (i.e., violating positive-definiteness of its elasticity tensor) is demonstrably thermodynamically unstable. Consequently, this new concept raises questions of material stability that are key to designing such novel materials.

STABILITY ANALYSIS

A comprehensive stability analysis has been performed to investigate the necessary and sufficient stability conditions for the elastic moduli of elastic composites with phases of negative incremental elastic moduli due to mechanical instabilities (such instabilities might arise, e.g., from structural buckling, prestressed members, or phase transformations). A key means to stabilizing negative stiffness is a sufficient geometric constraint enforced, e.g., by the stiff matrix phase surrounding the otherwise unstable inclusion phase.

We present results for various geometric model composites that prove that the inclusion phase may indeed violate positive-definiteness if embedded in a matrix that is sufficiently thick and sufficiently stiff. Examples comprise coated cylindrical and spherical inclusions as well as particle-matrix composites with a periodic microstructure, for which stability conditions are derived (and mutually confirmed) in several independent ways. First, (static, or energetic) sufficient stability conditions are obtained from Hill's energy approach [2, 3]. Second, the (kinematic, or dynamic) stability conditions are derived from a dynamic approach that identifies the infinite set of eigenfrequencies of the composite system and checks the range of constituent moduli that ensure stable eigenmodes [4] (i.e., real-valued eigenfrequencies for an elastic composite). Third, based on the energy approach, a relation between the stability condition and the overall elastic moduli of the composite can be derived which allows for the determination of closed-form analytical stability conditions. Finally, all obtained stability conditions are confirmed by finite element computations [5] that determine stability from a simple eigenvalue analysis of the overall stiffness matrix. For all cases investigated we present a complete stability map that identifies stable and unstable regimes of the elastic moduli of the inclusion and matrix phases.

For the model composites of coated cylinders and coated spheres, we show that the inclusion material may indeed violate positive-definiteness with the composite being stable overall. For isotropic media, we present closed-form analytical lower bounds on the inclusion-phase bulk modulus, which depends on the remaining elastic constants of both materials and the thickness of the coating: the stable regime grows with increasingly stiff and thick outer coatings. For a composite with periodic microstructure we demonstrate that, again, the matrix stabilizes non-positive-definite inclusion moduli with stability increasing with increasing matrix shear stiffness and decreasing with inclusion volume fraction. New analytical closed-form solutions are obtained for all cases and confirmed by numerical studies. Fig. 1 exemplarily illustrates the expanded stability regime of the inclusion phase in a coated-cylinder composite.

Finally, an interesting effect of stabilization due to gyroscopic forces will be reported. The investigation of the coated-

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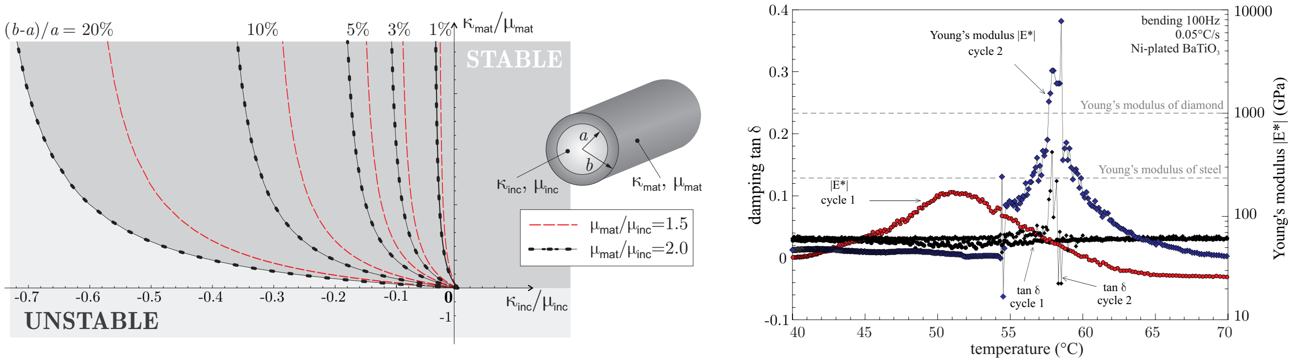

Figure 1. *Left:* expanded stability regime for the inclusion phase in a coated-cylinder composite, the stable regime increasing with increasing coating thickness and stiffness. *Right:* Experimental results for the dynamic Young modulus and loss tangent $\tan\delta$ of a tin-barium titanate composite vs. temperature, after [7].

cylinder composite under a constant axial rotation at high frequency reveals that the Coriolis acceleration present in the rotating elastic composite further stabilize a non-positive-definite inclusion phase [6] (i.e., negative incremental stiffness in the inclusion phase is stabilized not only by the constraining matrix phase but additionally by the rotation). This newly discovered effect has important implications on experimental realizations.

EXPERIMENTAL RESULTS

This novel material design concept is explored by embedding phases or components that exhibit mechanical instabilities into a stiff surrounding matrix. We will present successful examples for both structural and material systems. Structural samples comprise prestressed viscoelastic spring and truss systems whose snap-through and buckling behavior gives rise to an instability to be exploited for enhanced overall performance in the aforementioned way.

An impressive material application is the development of novel composites with extreme tunable behavior, and experimental results give evidence for bound-exceeding performance of a composite containing one phase with negative stiffness[7]. Barium titanate ($BaTiO_3$) is a ferroelectric ceramic that undergoes a structural transition near 120^o C, changing its lattice structure from cubic to tetragonal upon cooling. Governed by Landau's theory of phase transitions, the free energy landscape of bulk $BaTiO_3$ has one global minimum at temperatures above transition which changes into a local maximum below the transition temperature T_c. After cooling below T_c, the body (still in the high-temperature phase) is in an unstable equilibrium, resulting in a snap-through mechanism into the stable low-temperature configuration, which corresponds to the sought negative-stiffness effect. Samples of pure tin with 3 to 10 vol.-% of $BaTiO_3$ particles were prepared (using an ultrasonic casting techique) and tested by employing Broadband Viscoelastic Spectroscopy to measure the viscoelastic Young's modulus $|E^*|$ and loss tangent $\tan\delta$ at 100 Hz in bending and in torsion. It is conjectured that (as shown for the simple example of the rotating coated-cylinder composite [6]) the dynamic excitation of the sample during testing has a stabilizing effect on the stiffness and damping anomalies. Results show that the structural transition in the $BaTiO_3$ particles gives rise to extreme anomalies in the dynamic Young modulus and the damping of the overall composite (see Fig. 1), resulting in a Young modulus that temporarily exceeds that of diamond (ca. 1000 GPa) [7]. Such a strong increase in stiffness cannot be accomplished by means of positive-definite materials and is theorized to be due to a temporarily negative bulk modulus in the $BaTiO_3$ inclusions during transformation.

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