P-POSITIVE DEFINITE MATRICES AND DIVERGENCE STABILITY OF NON CONSERVATIVE CONSTRAINED SYSTEMS

J. Lerbet [*a)], M.Aldowaji [*], N. Challamel [**], F. Nicot [***]& F. Darve [****]
[*]*IBISC , UFRST-UEVE, 40, rue du Pelvoux CE 1455 91020 Evry Courcouronnes cedex, France*
[**]*Université Européenne de Bretagne Université de Bretagne Sud LIMATB UBS Lorient Centre de Recherche Rue de Saint Maudé BP 92116 56321 Lorient cedex, France*
[***]*Cemagref, ETNA Geomechanics Group, 2, rue de la papeterie 38042 St Martin d'Heres cedex , France*
[****]*Laboratoire Sols Solides Structures, UJF-INPG-CNRS, BP 53 38041 Grenoble cedex 9, France*

Summary Adding kinematic constraints to non-conservative systems subjected to circulatory forces leads to paradoxical effects similar to the well-known effects of destabilizing by adding damping. Divergence stability under such a family of p kinematic constraints leads to the concept of p-definite (non symmetric) matrices. The main result about the relationship with definite positive matrices is given shining in a new light the so-called second order work criterion concerning only the symmetric part of the stiffness matrix: this criterion is the optimal one for insuring the divergence stability of the system subjected to any family of kinematic constraints.

INTRODUCTION

In this paper,we are concerned by divergence instabilities of nonconservative n degree of freedom systems like elastic systems subjected to circulatory forces with additional kinematic constrains. It has recently been shown ([2]) that one additional constraint may have a paradoxical effect by softening or destabilizing the system: the critical divergence load of the constrained system may be lower than the critical divergence load of the free or unconstrained system and the so-called second order work criterion concerning only the symmetric part of the stiffness matrix may prevent this paradoxical effect: as far as the second order work criterion holds, one additional constraint cannot destabilize the free system. The generalization to any family of p constraints with $p < n$ leading to the new concept of p-positive definite matrix is investigated and proved.

SETTING OF THE PROBLEM

Divergence equation of a discrete free system Σ reads $K(\beta)X = 0$ where the stiffness matrix $K(\beta) \in \mathcal{M}_n(\mathbf{R})$ is generally a non-symmetric matrix (for example in case of non-conservative elastic systems as for circulatory systems), β denotes the loading parameter and $X = {}^t(x_1 \dots x_n)$ is the n dimensional perturbation of the equilibrium configuration. Adding p kinematic constraints leads to the similar following equation $K_A(\beta)Y = 0$ where $K_A(\beta) = \begin{pmatrix} K(\beta) & A \\ {}^tA & 0 \end{pmatrix}$, $Y = \begin{pmatrix} X \\ \Lambda \end{pmatrix}$ with $\Lambda = {}^t(\lambda_1 \dots \lambda_p)$ the Lagrange multiplier associated with the p constraints at the equilibrium configuration and $A \in \mathcal{M}_{np}(\mathbf{R})$ is the matrix of the linear p constraints. After some convenient calculations, divergence instabilities lead (see [3]) to look for matrices $B \in \mathcal{M}_{np}(\mathbf{R})$ such that $\det({}^tBK(\beta)B) = 0$.

P-DEFINITE POSITIVE MATRICES

Definition An element K of $\mathcal{M}_n(\mathbf{R})$ is said to be p-positive definite iff $\det({}^tBKB) > 0$ for all matrix $B \in \mathcal{M}_{np}(\mathbf{R})$ with maximal rank p.
For several and general mathematical properties of the p-positive definite matrices see [3]. Remark that for $p = 1$ an 1-positive definite matrix K is only a positive definite matrix involving only its symmetric part K_s: a matrix K is 1-positive definite if and only if the spectrum of its symmetric part K_s is included in $\mathbf{R}_+^*$. For p-positive definite matrices, the main result is:
Theorem 1 If K is 1-positive definite then for any $1 \le p < n$, K is p-positive definite. (for the proof, see [3])
Concerning mechanical meaning of p-positive definite matrices, the main consequence is:
Theorem 2 Suppose the free system Σ is divergence stable. In order to this system remains divergence stable by adding p or less than p constraints, it is necessary and sufficient that the second order work criterion holds: as long as the symmetric part of the stiffness matrix remains definite positive, no additional systems of constraints may destabilize the system. Moreover, for a loading parameter β_s such that $\det(K_s(\beta_s)) = 0$, there is a family of p constraints destabilizing the system and the proof is constructive for finding a family of contraints.

[a)] Corresponding author. E-mail: jean.lerbet@iup.univ-evry.fr

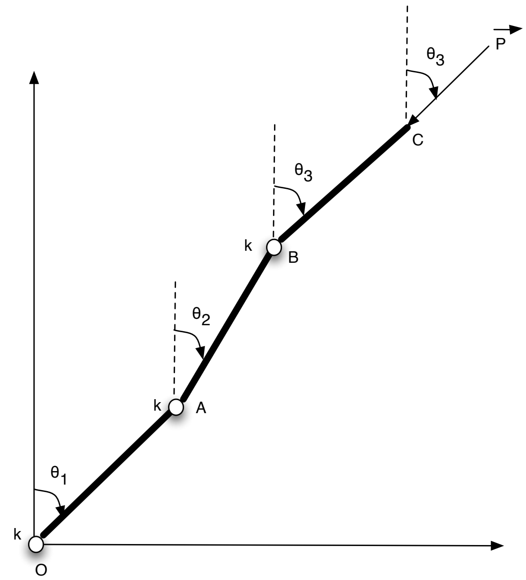

Figure 1. 3 dof Ziegler system

AN ILLUSTRATIVE EXAMPLE

Consider a three degree of freedom $(\theta_1, \theta_2, \theta_3)$ Ziegler's column (figure 1) subjected to a complete follower force $\vec{P}$. The dimensionless loading parameter is $\beta = \frac{P\ell}{k}$ where k is the sitffness of the joints and ℓ the length of each bar. The equilibrium configuration is the vertical one $(0,0,0)$. Calculations give $K(\beta) = \begin{pmatrix} 2-\beta & -1 & \beta \\ -1 & 2-\beta & -1+\beta \\ 0 & -1 & 1 \end{pmatrix}$. Because $\det(K(\beta)) = 1$, the free system is always divergence stable. The roots of $\det(K_s(\beta)) = 0$ are $1, 1-\sqrt{3}, 1+\sqrt{3}$. Applying the previous result of theorem 2, as long as $\beta < 1$, no system of one or two constraints may destabilize the system. For $\beta = 1$, the algebraic procedure deduced from the proof of theorem 1 leads to choose the matrix $B = \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}$. Thus ${}^tBK(1)B = 0_{\mathcal{M}_2(\mathrm{R})}$ and obviously $\det({}^tBK(1)B) = 0$. The corresponding family of constraints is $A = K(1)B$ which leads to the matrix $\begin{pmatrix} 0 & 1 \\ 0 & -1 \\ -1 & 0 \end{pmatrix}$ meaning that the 3 dof Ziegler system constrained by $\theta_1 - \theta_2 = 0$ and $-\theta_3 = 0$ is divergence unstable for $\beta = 1$ which may be immediately checked.

CONCLUSION

By using the new concept of p-positive definite matrix and the main result of theorem 1 about this class of matrices, we have claimed and illustrated that the second-order work criterion is the optimum criterion for a divergence stable system for remaining divergence stable if subjected to any family of kinematical constraints. The proofs may be find in [3].

References

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