BUCKLING OF FLEXIBLE ELECTRONIC STRUCTURE IN ASYMMETRIC CHECKERBOARD MODE

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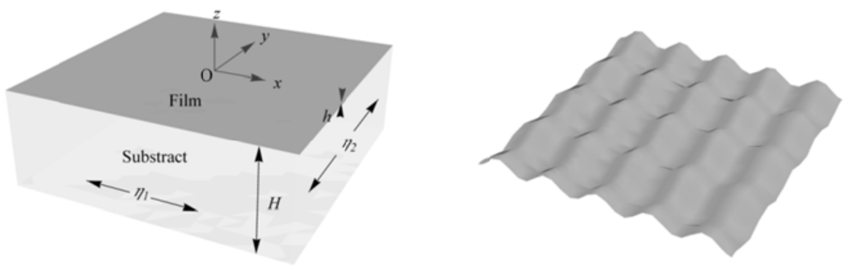
Summary The flexible electronic structure buckles into a checkerboard mode under biaxial prestrain. Moreover, the secondary bifurcation of this structure occurs in a special window of prestrain from a symmetric checkerboard mode to an asymmetric one. The buckling modes and the critical condition under arbitrarily prestrain are given in this paper by energy method.

INTRODUCTION

The flexible electronic structures have important applications in microelectronics, biology, medicine and military. It is based on the buckling of a stiff film bound to a compliant substrate. Chen and Hutchinson [1] study various buckling modes under biaxial prestrain including straight, checkerboard and herringbone. And the herringbone has the lowest energy which can be observed in experiments probably. Audoly and Boudaoud [2] point out the secondary instabilities of these modes and present a weakly nonlinear post-buckling analysis. They find that the square checkerboard mode is optimal just above the threshold under equi-biaxial prestrain. The straight mode becomes unstable above a primary threshold under anisotropic prestrain. S. Cai et. al [3] employ two methods to ascertain the energy in buckling. For flat films, the checkerboard mode is preferred only above the threshold. Except that the herringbone mode has the lowest energy. In this paper, we focus on the case of anisotropic prestrain near the region of equi-biaxial. The asymmetric checkerboard mode is analyzed. The buckling modes (amplitude and wave number) and the critical condition under arbitrarily prestrain are given by energy method. The secondary bifurcation of checkerboard mode occurs from symmetric to asymmetric mode.

FORMULATION

A stiff elastic thin film is bonded to a compliant elastic thick substrate without slipping. The substrate is imposed biaxial prestrains η_1 and η_2, while the film is free. A coordinate system is established in this configuration with an origin at the interface, as Fig. 1a. After releasing the prestrain in the substrate, the film buckles into a checkerboard mode, as Fig. 1b. The substrate is described by small deformation theory, while the film is described by the von Karman thin plate theory undergoing moderate deflections. The tangential displacement at the interface is assumed zero [2] and the bottom of the substrate is fixed. The deflection of the film is assumed as $w=A_1\cos(k_1x)+A_2\cos(k_2y)$.



(a) (b)

Fig. 1 The film/substrate structure and a checkerboard mode

The energy of the structure consists of the bending and stretching energy of the film and the strain energy of the substrate. The prestrain, wave number, amplitude and energy are rescaled by $\eta_i=\eta_0\bar{\eta}_i, k_i=k_0\bar{k}_i, A_i=\bar{A}_i h, \Pi=\bar{\Pi}\Pi_0$ (i=1,2), where the parameters and effective modulus are

$$\eta_0=\frac{1}{(1+\nu_f)}\left(\frac{3E_s^*}{2E_f^*}\right)^{2/3},\ k_0=\frac{1}{h}\sqrt[3]{\frac{12E_s^*}{E_f^*}},\ \Pi_0=3E_s^*h\left(\frac{3E_s^*}{2E_f^*}\right)^{2/3},E_f^*=\frac{E_f}{1-\nu_f^2},\ E_s^*=\frac{E_s(1-\nu_s)}{3-\nu_s-4\nu_s^2} \tag{1}$$

The Young's modulus of the film and substrate are E_f and E_s, while the Poisson's ratios are ν_f and ν_s. The energy of the structure is $\bar{\Pi}=\bar{\alpha}+\bar{\beta}_1\bar{A}_1^2+\bar{\beta}_2\bar{A}_2^2+\gamma_0\bar{A}_1^2\bar{A}_2^2+\gamma_1\bar{A}_1^4+\gamma_2\bar{A}_2^4$ with the parameters

$$\bar{\alpha}=\frac{\bar{\eta}_1^2+2\bar{\eta}_2\bar{\eta}_1\nu_f+\bar{\eta}_2^2}{2(1+\nu_f)^2},\ \bar{\beta}_1=\frac{\bar{k}_1(\bar{k}_1^3+2)}{3}-\frac{\bar{k}_1^2(\bar{\eta}_1+\nu_f\bar{\eta}_2)}{1+\nu_f},\ \bar{\gamma}_0=\bar{k}_1^2\bar{k}_2^2\left[\nu_f+\frac{2\bar{k}_1^2\bar{k}_2^2(1-\nu_f^2)}{(\bar{k}_1^2+\bar{k}_2^2)^2}\right],\ \bar{\gamma}_1=\frac{\bar{k}_1^4}{2} \tag{2}$$

$\bar{\beta}_2$ and $\bar{\gamma}_2$ are obtained by the rotation of the subscripts 1,2. In order to find the equilibrium amplitude and wave number, we set $\partial\bar{\Pi}/\partial\bar{A}_i=\partial\bar{\Pi}/\partial\bar{k}_i=0$ and get the nonlinear equations

$$\begin{cases}2+(1+3\bar{A}_1^2)\bar{k}_1^3-3(\bar{\eta}_1+\nu_f\bar{\eta}_2)\bar{k}_1/(1+\nu_f)+3\bar{A}_2^2\bar{k}_1\bar{k}_2^2\left[\nu_f+2\bar{k}_1^2\bar{k}_2^2(1-\nu_f^2)/(\bar{k}_1^2+\bar{k}_2^2)^2\right]=0\\ 1+(2+3\bar{A}_1^2)\bar{k}_1^3-3(\bar{\eta}_1+\nu_f\bar{\eta}_2)\bar{k}_1/(1+\nu_f)+3\bar{A}_2^2\bar{k}_1\bar{k}_2^2\left[\nu_f+4\bar{k}_1^2\bar{k}_2^2(1-\nu_f^2)/(\bar{k}_1^2+\bar{k}_2^2)^3\right]=0\end{cases} \tag{3}$$

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Similarly, the other two equations are obtained by the rotation of subscripts. As the symmetry, it is assumed that $\bar{\eta}_1 \geq \bar{\eta}_2$. If the wavelength is isotropic, there must be $\bar{k}_1 = \bar{k}_2 = 1$. And the amplitudes are

$$\bar{A}_{\mathrm{eq1}} = \sqrt{\frac{2\left[(2-v_{\mathrm{f}})\bar{\eta}_1 - \bar{\eta}_2 - 1 + v_{\mathrm{f}}\right]}{3 - v_{\mathrm{f}} - 3v_{\mathrm{f}}^2 + v_{\mathrm{f}}^3}}, \quad \bar{A}_{\mathrm{eq2}} = \sqrt{\frac{2\left[(2-v_{\mathrm{f}})\bar{\eta}_2 - \bar{\eta}_1 - 1 + v_{\mathrm{f}}\right]}{3 - v_{\mathrm{f}} - 3v_{\mathrm{f}}^2 + v_{\mathrm{f}}^3}} \tag{4}$$

If we set $\bar{A}_{\mathrm{eq2}} = 0$, the critical condition is deduced as $1 - v_{\mathrm{f}} + \bar{\eta}_{\mathrm{cr1}} - (2 - v_{\mathrm{f}})\bar{\eta}_{\mathrm{cr2}} = 0$. If the wavelength is anisotropy, a ratio of the wave numbers is introduced, $q = \bar{k}_2 / \bar{k}_1$. The amplitudes are in term of q and $\bar{k}_1$,

$$\bar{A}_{\mathrm{eq1}} = \sqrt{\frac{(1+q^2)^3(\bar{k}_1^3 q^3 - 1)}{6(1-v_{\mathrm{f}}^2)(q^2-1)\bar{k}_1^3 q^3}}, \bar{A}_{\mathrm{eq2}} = \sqrt{\frac{(1+q^2)^3(\bar{k}_1^3 - 1)}{6(1-v_{\mathrm{f}}^2)(q^2-1)\bar{k}_1^3 q^4}} \tag{5}$$

The implicit functions of the wave numbers and prestrains are

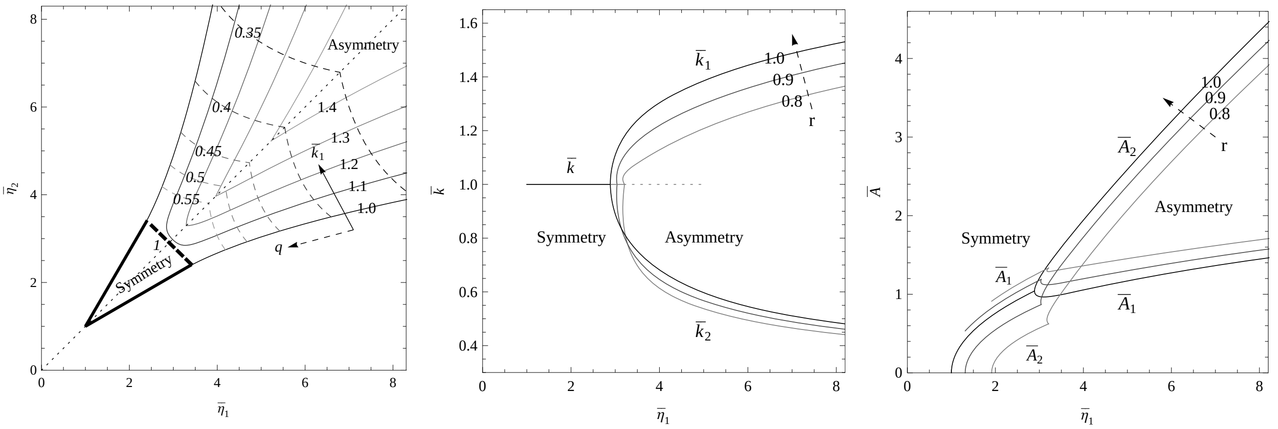
$$\begin{cases} 6(1-v_{\mathrm{f}})(1-q^2)q^3\bar{k}_1\bar{\eta}_1 = \left[3 - 3q^2 - (3-4v_{\mathrm{f}})q^4 - q^6\right]q^3\bar{k}_1^3 + \left[1 + (3-6v_{\mathrm{f}})q^2 + 2q^3 + (3+2v_{\mathrm{f}})q^4 - 6q^5 + q^6\right] \\ 6(1-v_{\mathrm{f}})(1-q^2)q^2\bar{k}_1\bar{\eta}_2 = \left[1 + (3-4v_{\mathrm{f}})q^2 + 3q^4 - 3q^6\right]\bar{k}_1^3 - \left[1 - 6q + (3+2v_{\mathrm{f}})q^2 + 2q^3 + (3-6v_{\mathrm{f}})q^4 + q^6\right] \end{cases} \tag{6}$$

The critical condition of the asymmetric checkerboard mode is deduced by setting $\bar{k}_1 \to 1$, which is a parametric form of q. And the boundary between two types of checkerboard modes is deduced by setting $q \to 1$: $\bar{\eta}_{\mathrm{cr1}} + \bar{\eta}_{\mathrm{cr2}} = 2/(1-v_{\mathrm{f}}) + 3$.

For any prestrain satisfied the critical condition, the wave number and amplitude are solved according to Eqs.(4) ~(6). And then the buckling mode and minimum energy are determined.

DISCUSSION

Fig.2a shows the relationship among the wave number $\bar{k}_1$, q and prestrain $\bar{\eta}_i$. The solid and the dashed line (italic number) are the contours of $\bar{k}_1$ and q, respectively. The outer line ($\bar{k}_1 = 1$) indicates the critical condition. In the region surrounding by bold lines (q=1), the film buckles into a symmetry checkerboard mode with isotropic wavelength, although the amplitudes may be anisotropy when the prestrain is anisotropy. In the remaining region surrounding by thin lines (q<1), the film buckles into an asymmetry checkerboard mode with anisotropy wavelength. Fig.2b,c show the relationships between wave numbers, amplitudes and prestrain, where the ratio $r = \bar{\eta}_2 / \bar{\eta}_1$. The wave numbers and amplitudes both buckle twice. In the secondary buckling, the wave numbers branch, while the amplitudes deflect.



(a) (b) (c)

Fig. 1 The critical condition and post-buckling of the checkerboard mode

CONCLUSION

The flexible electronic structure buckles into a checkerboard mode under biaxial prestrain. The secondary bifurcation of the structure occurs from a symmetric checkerboard mode to an asymmetric one. The checkerboard modes and the critical conditions under arbitrarily prestrain are given. In fact, the biaxial buckling modes are various. It needs to compare the energy of different modes.

Reference

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