REDUCED STIFFNESS CRITICAL PRESSURE OF SPHERICAL SHELLS HAVING LOCALISED IMPERFECTIONS

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Summary The reduced stiffness analysis for imperfection-sensitive spherical shell domes under external pressure is carried out. The analytical models are for both orthotropic and isotropic material formulations, and a new theoretical idea gives very simple solutions. It has been shown that the critical pressure decrease as the wave length of reduced stiffness buckling associated with that of localised single-dimple imperfection becomes longer. A nonlinear finite element analytical result for isotropic shells shows similar tendency to that of the present solution.

INTRODUCTION

For shallow spherical shell caps [1,2] some success has been achieved in obtaining good agreement between experiment and theory: Yamada and co-workers [3,4] provided the first detailed resolution of the discrepancies between experiment and theory, and nonlinear FEM simulations [5] for shells having recorded initial imperfection shapes[6]. Using mixed variational finite element analysis with nine-node-shell elements, it was shown that not only buckling loads but also pre- and post-buckling displacement distributions could be produced [7].

For the design of this kind of imperfection-sensitive buckling problem, the non-linear analytical approach using a deterministic "worst" imperfection mode [8] or the linear "reduced stiffness" approach with no-necessity for any directly defined imperfections [9,10], have been proposed. In this paper, most simplest lower limit criteria for the elastic buckling of orthotropic or isotropic spherical shells under external pressure have been represented.

REDUCED STIFFNESS BUCKLING ANALYSIS FOR ORTHTROPIC SPHERICAL SHELL DOMES

The fundamental stress for a thin-walled partial spherical dome with radius R under pressure p is in the idealised state given by

$$n_x^E = -pR/2, \quad n_y^E = -pR/2, \quad n_{xy}^E = 0 \tag{1}$$

The present simplified reduced stiffness analysis in which the analytical region is limited and concentrated on only a typical lobe of the incremental deformed area associated with the lower bound criteria for moderately large imperfections.

This treatment is very similar to those which have been already developed in Ref. [11,12] discussing the local buckling of plates on elastic foundations. For a truncated shallow dome shown in Fig.1, the buckling displacements for half-wave lengths l_x and l_y could be approximately adopted as

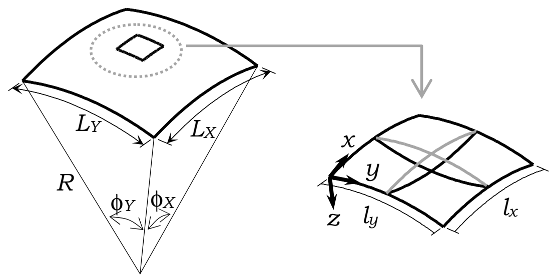

Fig.1 Critical lobe model of spherical shell dome

$$u^d = u_1 \cos\frac{\pi x}{l_x}\sin\frac{\pi y}{l_y}, \qquad v^d = v_1 \sin\frac{\pi x}{l_x}\cos\frac{\pi y}{l_y}, \qquad w^d = w_1 \sin\frac{\pi x}{l_x}\sin\frac{\pi y}{l_y} \tag{2}$$

The change in the total potential energy may be written as

$$\delta\left(U_{2b} + U_{2m} + V_{2m}\right) = 0 \tag{3}$$

where U_{2b} and U_{2m} are respectively the quadratic components of the bending and membrane strain energies, V_{2m} is the quadratic part of the interactions between the idealised prebuckling state and the nonlinear membrane stresses and strains associated with the buckling displacements written in Eq.2.

The Donnell-type approximations for the relationships between the incremental strains and the buckling displacements are adopted; the constitutive equations for the orthogonal domes are

$$\begin{aligned} m_x^d &= D_{11}\kappa_x^d + D_{12}\kappa_y^d, \quad & m_y^d &= D_{12}\kappa_x^d + D_{22}\kappa_y^d, \quad & m_{xy}^d &= 2D_{66}\kappa_{xy}^d \\ n_x^d &= A_{11}\varepsilon_x^d + A_{12}\varepsilon_y^d, \quad & n_y^d &= A_{12}\varepsilon_x^d + A_{22}\varepsilon_y^d, \quad & n_{xy}^d &= 2A_{66}\varepsilon_{xy}^d \\ n_x^{dd} &= A_{11}\varepsilon_x^{dd} + A_{12}\varepsilon_y^{dd}, \quad & n_y^{dd} &= A_{12}\varepsilon_x^{dd} + A_{22}\varepsilon_y^{dd}, \quad & n_{xy}^{dd} &= 2A_{66}\varepsilon_{xy}^{dd} \end{aligned} \tag{4}$$

where (D_{11}, D_{12}, D_{22}, D_{66}) are bending stiffness components, and (A_{11}, A_{12}, A_{22}, A_{66}) membrane stiffness components.

Consequently the full stiffness buckling load p^C resulting from the solution of the eigenvalue equations takes the form $p^C = p^b + p^m$ in which p^b is the bending component and p^m the membrane component.

A number of recent studies [9, 13] have shown that for imperfection sensitive buckling of shells, the reduced stiffness buckling load p^* gives a rational lower bound criterion for design. In the reduced stiffness method the theoretical lower bound membrane stiffness components, which contribute to the loss of load-carrying capacity in the postbuckling response, are eliminated. It is a fairly straight forward matter to identify in the mode characterising the full stiffness buckling load the energy components which are the risk through the erosive effects of mode coupling catalysed by the presence of initial imperfections. In the present problem the reduced stiffness buckling load is obtained as

$$p^* = p^b = 2\left\{D_{11}\xi_x^4 + 2\left(D_{12} + 2D_{66}\right)\xi_x^2\xi_y^2 + D_{22}\xi_y^4\right\} / \left\{R\left(\xi_x^2 + \xi_y^2\right)\right\} \quad (5)$$

where $\xi_x \equiv \pi/l_x$ and $\xi_y \equiv \pi/l_y$.

FOR ISOTROPIC SPHERICAL SHELL DOMES

For an isotropic material with Young's modulus E and Poisson's ratio ν , the components p^b and p^m are obtained as as

$$p^C = p^b + p^m, \qquad p^b = p^* = 2D\left(\xi_x^2 + \xi_y^2\right)/R \quad \text{and} \quad p^m = 2K\left(1-\nu^2\right)/\left\{R^3\left(\xi_x^2 + \xi_y^2\right)\right\} \quad (6)$$

where $D \equiv Et^3/\left\{12\left(1-\nu^2\right)\right\}$, $K \equiv Et/\left(1-\nu^2\right)$ and t = thickness. Now $(D_{11}, D_{12}, D_{22}, D_{66}) = \{D, D\nu, D, D(1-\nu)/2\}$ and $(A_{11}, A_{12}, A_{22}, A_{66}) = \{K, K\nu, K, K(1-\nu)/2\}$ in Eq.4. have been adopted. The minimisation of p^C gives the well-known classical buckling pressure of spherical shells $p_{cl} = 4\sqrt{\left(1-\nu^2\right)DK}/R^2 = 2Et^2/\left\{R^2\sqrt{3\left(1-\nu^2\right)}\right\}$ and the circle equation $\xi_x^2 + \xi_y^2 = \sqrt{\left(1-\nu^2\right)\left(K/D\right)}/R$ for l_x and l_y; this circle equation is similar to "Koiter's circle" for axially compressed cylinders.

In the case of an isotropic spherical shell cap having a circular plan, base radius a and geometrical parameter $\lambda \equiv \left\{12\left(1-\nu^2\right)\right\}^{1/4} a/\sqrt{Rt}$, the reduced stiffness pressure is obtained from Eq.6 using $l_x = l_y = l$ as

$$p^* = p_{cl}\left\{\pi a/(\lambda l)\right\}^2 \quad (7)$$

The precise non-linear FEM results [10] for non-shallow spherical shells having localised single-dimple imperfections a = 200 mm, R = 540 mm, t = 0.9 mm and λ =16.3 have given the ratio of the dimple radius to the base radius r/a =0.235 for the lower bound $p_{\mathrm{FEM}}/p_{cl} \approx 0.2$ in the case of the imperfection amplitude equal to shell thickness. It will be confirmed that $(l/2)/a$ =0.215 through Eq.7 gives $p^*/p_{cl} = 0.2$. In spite of the difference between the present buckling wave-length and the radius of dimple imperfection, the good agreement suggests that the present simplified reduced stiffness criteria provide a rational and safe prediction for the design of pressure loaded spherical shells.

CONCLUSIONS

Based upon the previous fruits using physical buckling experiments, their non-linear simulations considering imperfections and the innovative reduced stiffness analysis for pressure loaded spherical shell domes, this paper proposes a general and simplified criterion on the elastic lower-bound buckling for orthotropic and isotropic materials. The quality of the results is enhanced by the comparison of the proposed equation and some of the published FEM results. Additional confirmations for various geometries and boundary conditions would be needed in the future.

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