# MICROSCOPIC AND MACROSCOPIC INSTABILITIES IN FIBER-REINFORCED ELASTOMERS

Oscar Lopez-Pamies[*a)], Jean-Claude Michel[**], Pedro Ponte Castañeda[***], Nicholas Triantafyllidis[****]
[*]*Department of Civil and Environmental Engineering,*
*University of Illinois at Urbana–Champaign, IL 61801-2352, USA*
[**]*Laboratory of Mechanics and Acoustics, Marseille 13402,France*
[***]*Department of Applied Mechanics, University of Pennsylvania, Philadelphia, PA 19104–6315, USA*
[****]*Laboratory of Solid Mechanics & Department of Mechanics,*
*École Polytechnique, 91128 Palaiseau Cedex, France*

*Summary* The present work is a detailed study of the connections between microstructural instabilities and their macroscopic manifestations — as captured through the effective properties — in finitely strained fiber-reinforced elastomers, subjected to finite, plane-strain deformations normal to the fiber direction. The work, which is a complement to a previous and analogous investigation by the same authors on porous elastomers, (Michel et al., 2007), uses a linear comparison technique, initially developed for random media, to study the onset of failure in periodic fiber-reinforced elastomers and to compare the results to more accurate finite element method (FEM) calculations. The influence of different fiber distributions (random and periodic), initial fiber volume fraction, matrix constitutive law and fiber cross-section on the microscopic buckling (for periodic microgeometries) and macroscopic loss of ellipticity (for all microgeometries) is investigated in detail. In addition, constraints to the principal solution due to fiber/matrix interface decohesion, matrix cavitation and fiber contact are also addressed. It is found that both microscopic and macroscopic instabilities can occur for periodic microstructures, due to a symmetry breaking in the periodic arrangement of the fibers. On the other hand, no instabilities are found for the case of random microstructures with circular section fibers, while only macroscopic instabilities are found for the case of elliptical section fibers, due to a symmetry breaking in their orientation.

## INTRODUCTION

Failure in composite materials is a fundamental as well as an extremely diverse issue in solid mechanics. Questions about what constitutes failure, when does failure start and whether it is possible to predict the onset of failure by investigating the effective (homogenized) properties of the solid, are problems of fundamental interest for all composites. The infinite variety of matrix materials and microstructures, which result in a multitude of possible failure mechanisms, is the reason for the problem's extraordinary diversity. In the interest of (relative) simplicity, and motivated by considerable recent advances in calculating the effective properties of finitely strained nonlinear solids, attention is focused here on particle-reinforced elastomers. To further simplify this complex problem, only two-dimensional such elastomers are considered here, which can also be viewed as fiber-reinforced elastomers under in-plane loading perpendicular to their fiber axes. Even this particular class of composites is of considerable practical interest and enjoys a wide range of technological applications, e.g., car tires, many types of biological tissues, and block copolymers with cylindrical microstructures, just to name a few.

To relate the onset of failure in these composites to their effective properties, one must address the issues of homogenization and of microstructure stability. On the homogenization front, a considerable effort has been devoted over the last 40 years to predict the effective properties of hyperelastic composites with random microstructures. From the increasingly accurate homogenization schemes which have been devised, of particular interest here is the linear comparison method of Lopez-Pamies and Ponte Castañeda (2006). This technique is sophisticated enough to account for the evolving microstructure and is thus capable of predicting a macroscopic material failure in the form of a loss of strong ellipticity (or rank-one convexity) of the corresponding homogenized behavior.

On the microstructural stability side, Geymonat, Müller, and Triantafyllidis (1993) established a rigorous connection between bifurcation instability at the microscopic level and loss of rank-one convexity of the homogenized moduli in finitely strained periodic elastomers of infinite extent. More specifically, it was shown that if the wavelength of the bifurcation eigenmode is infinite (compared to the unit cell size), the corresponding instability of the periodic principal solution can be detected as a loss of ellipticity of the corresponding one-cell homogenized tangent moduli of the solid. Based on these general results, onset-of-failure surfaces in stress and strain space can be readily defined for periodic solids of infinite extent.

The present work is a complement to a recent, comprehensive investigation by the same authors on porous elastomers (Michel et al., 2007), and provides a detailed study of the connections between microstructural instabilities and their macroscopic manifestations — as captured through the effective properties — in finitely strained fiber-reinforced elastomers, subjected to finite, plane-strain deformations normal to the fiber direction. The work uses the above-mentioned linear comparison technique, to study the onset of failure in periodic fiber-reinforced elastomers and to compare the results to more accurate finite element method (FEM) calculations. The influence of different fiber distributions (random and periodic), initial fiber volume fraction, matrix constitutive law and fiber cross-section on the microscopic buckling (for periodic microgeometries) and macroscopic loss of ellipticity (for all microgeometries) is investigated in detail. In addition, constraints to the principal solution due to fiber/matrix interface decohesion, matrix cavitation, and fiber contact

[a)] Corresponding author. E-mail: pamies@illinois.edu

are also addressed, thus giving a complete picture of the different possible failure mechanisms present in this class of elastomeric composites.

## SAMPLE RESULTS

For demonstration purposes, Figure 1 shows onset-of-failure curves for a Neo-Hookean solid reinforced by a hexagonal distribution of circular rigid fibers with volume fraction $f_0 = 20\%$. The results are presented in the spaces of deformation gradients (part a) and first Piola-Kirchhoff stresses (part b) for the loss of strong ellipticity, onset of first bifurcation, matrix cavitation, fiber/matrix decohesion, and fiber contact.

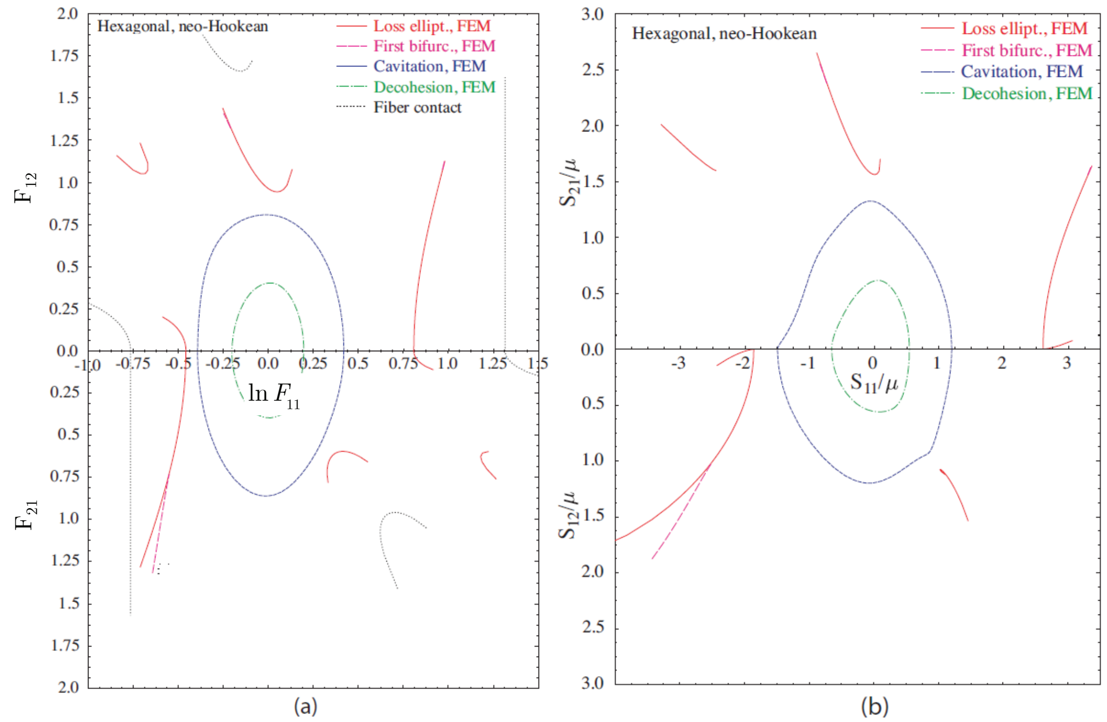


**Figure 1.** Onset-of-failure curves for a Neo-Hookean solid reinforced by a hexagonal distribution of circular rigid fibers with volume fraction $f_0 = 20\%$, which is subjected to isochoric plane strain loading; (a) results in macroscopic strain space and (b) in dimensionless macroscopic stress space. Macroscopic loss-of-ellipticity is depicted by solid lines, onset of first bifurcation by big dash lines, matrix cavitation by small dash lines, fiber decohesion by dot-dash lines and fiber contact by dotted lines. Results are based on FEM calculations.

## References


[1] Geymonat, G., Müller, S. and Triantafyllidis, N.: Homogenization of Nonlinearly Elastic Materials, Microscopic Bifurcation and Macroscopic Loss of Rank-One Convexity. *Arch. Rat. Mech. Anal.* 122 (1993), 231–290.

[2] Lopez-Pamies, O., Ponte Castañeda, P.: On the overall behavior, microstructure evolution, and macroscopic stability in reinforced rubbers at large deformations: I — Theory. *Journal of the Mechanics and Physics of Solids* 54 (2006a), 807–830.

[3] Michel, J.C., Lopez-Pamies, O., Ponte Castañeda, P., Triantafyllidis, N.: Microscopic and macroscopic instabilities in finitely strained fiber-reinforced elastomers. Journal of the Mechanics and Physics of Solids 58 (2010), 1776–1803.

[4] Michel, J.C., Lopez-Pamies, O., Ponte Castañeda, P., Triantafyllidis, N.: Microscopic and macroscopic instabilities in finitely strained porous elastomers. *Journal of the Mechanics and Physics of Solids* 55 (2007), 900–938.