STRETCH-INDUCED WRINKLING OF HYPERELASTIC THIN SHEETS

Vishal Nayyar, K. Ravi-Chandar & Rui Huang [a)]
Department of Aerospace Engineering and Engineering Mechanics, The University of Texas at Austin, Austin, Texas 78712, USA

Summary Wrinkles are commonly observed in stretched thin sheets and membranes. This paper presents a numerical/experimental study on stretch-induced wrinkling of hyperelastic thin sheets. A two-dimensional stress analysis is performed for a rectangular sheet under uniaxial stretching with two clamped ends and two free edges, followed by buckling and post-buckling analyses. Experimental measurements with polyethylene sheets are presented in comparison with the numerical results.

INTRODUCTION

Thin sheets and membrane structures are used widely in space applications such as inflatable space antennas, solar sails, and radars. Surface flatness over a large area is one of the key requirements in many of these applications. It has been shown that wrinkles often appear in thin sheets under a variety of loading conditions [1-3]. It is thus important to understand the mechanics of wrinkling for reliable control of surface wrinkles. In the present study we consider a rectangular sheet subjected to uniaxial stretch with two opposite ends clamped (Fig. 1). Friedl et al. [4] showed that compressive transverse stresses are induced in a linear elastic sheet upon stretching. Jacques and Potier-Ferry [5] presented an analytical solution to explain the wrinkle wavelength selection and mode localization in relatively long sheets. Cerda et al. [2] presented the first experimental data for stretch-induced wrinkle wavelengths along with a scaling analysis. Recently, Puntel et al. [6] confirmed the scaling relation for the wrinkle wavelength but showed that wrinkling occurs only beyond a critical stretch. Zheng [7] presented both numerical analyses and experimental measurements of the wrinkles. In the present study, we start with a two-dimensional analysis of stress distribution, which predicts various patterns of stretch-induced compressive stresses for the rectangular sheets with different length-to-width aspect ratios. Subsequently, we present linear eigenvalue analyses and nonlinear post-buckling analyses to simulate evolution of stretch-induced wrinkles. Finally, experimental measurements with polyethylene sheets are presented.

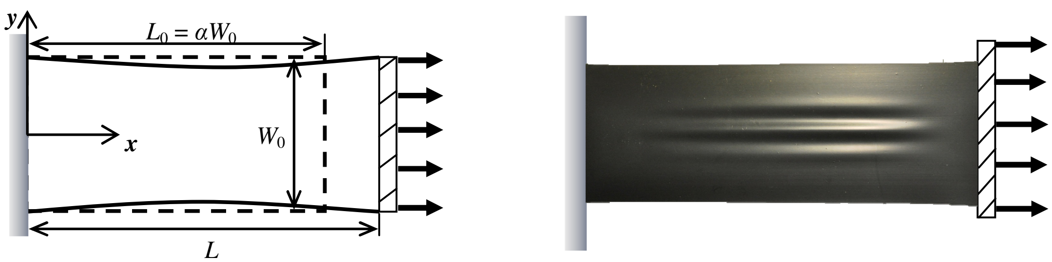

Figure 1. Left: Schematic illustration of a rectangular sheet with two clamped-ends, subject to uniaxial stretch. Right: An optical image of a wrinkled polyethylene sheet under stretch (ε ~ 10%).

TWO-DIMENSIONAL STRESS ANALYSIS

Before stretching, the dimensions of the sheet are: length L_0, width W_0, and thickness t_0. Define the nominal strain as $\varepsilon = L/L_0 - 1$, where L is the end-to-end distance of the stretched sheet. In this section, we present a two-dimensional (2-D) stress analysis, assuming no wrinkles. The geometry of the rectangular sheet is characterized by a single dimensionless ratio, $\alpha = L_0/W_0$. Note that the thickness of the sheet is important only for the wrinkling analysis. An incompressible neo-Hookean model with the initial shear modulus μ = 6 MPa is used in the analysis with nonlinear kinematics. For each aspect ratio, the sheet is increasingly stretched by prescribing the end displacements. It is found that for $\alpha > 1$ compressive stresses are induced in the transverse direction of the sheet (i.e., $\sigma_y < 0$), a necessary condition for wrinkling. Different distribution patterns of the stretch-induced compressive stress are predicted [8], summarized in a phase diagram (Fig. 2a) with four different phases: (I) all-tension phase; (II) transverse two-peak phase; (III) central one-peak phase; (IV) longitudinal two-peak phase. Furthermore, the magnitude of the peak compressive stress first increases with the nominal strain and then decreases (Fig. 2b). The maximum magnitude of the peak compressive stress increases with the aspect ratio for $1 < \alpha < 1.8$, but decreases for $\alpha > 1.8$ (Fig. 2c). For high aspect ratios ($\alpha > 3.7$), the peak compressive stress and its dependence on the nominal strain becomes independent of the aspect ratio.

BUCKLING ANALYSIS

We perform an eigenvalue buckling analysis for pre-stretched sheets, followed by a nonlinear post-buckling analysis. A small pre-stretch of 1% is applied for the eigenvalue analysis. For $\alpha \geq 1.35$, a number of eigenmodes are obtained. For

[a)] Corresponding author. Email: ruihuang@mail.utexas.edu.

the post-buckling analysis, a small geometrical imperfection is introduced by using the eigenmodes. For a rectangular sheet with $\alpha = 2.5$ and $W_0/t_0 = 1000$, the out-of-plane displacement (u_z) along the mid-section ($x = L/2$) is plotted in Fig. 3a. The wrinkle amplitude grows significantly from $\varepsilon = 5\%$ to $\varepsilon = 10\%$, but then decreases with increasing strain and eventually the sheet is nearly flattened at $\varepsilon = 30\%$. Similar results are obtained for different aspect ratios, as shown in Fig. 3b. Such a wrinkling behavior is in contrast with the scaling analysis by Cerda et al. [2], which predicted the wrinkle wrinkle amplitude to increase monotonically with increasing strain. Figure 3a shows that the sheet remains flat near the free edges and the wrinkling profiles are modulated by a long-wavelength mode, similar to the prediction by Puntel et al. [6]. However, we note that the number of wrinkles in the sheet remains nearly a constant. On the other hand, the wrinkle wavelength does decrease with increasing strain, in agreement with the scaling analysis by Cerda et al. [2]. It may thus be concluded that the scaling analysis offers a satisfactory prediction of the wrinkle wavelength, but less so for the wrinkle amplitude.

EXPERIMENTS

For the experiments, thin polyethylene sheets of different aspect ratios and thicknesses were used. Each sheet was stretched at a rate of 1 in/min under displacement control up to 40% nominal strain. The experimental results confirmed formation of stretch induced wrinkles, which were measured by digital image correlation technique. For a thin sheet with thickness $t_0 \sim 0.1$ mm and an aspect ratio $\alpha = 2.5$ ($L_0 = 250$ mm and $W_0 = 100$ mm), the experiment confirmed non-monotonic evolution of the wrinkle amplitude as predicted by numerical simulations. However, the measured wrinkle amplitude is considerably greater than the prediction. On the other hand, the wrinkle wavelength is in good agreement with the simulation. It was noted that the material underwent plastic deformation in the experiment. As a result, the wrinkles were not flattened at large strains. More experiments and analyses are underway to better understand the phenomenon.

References

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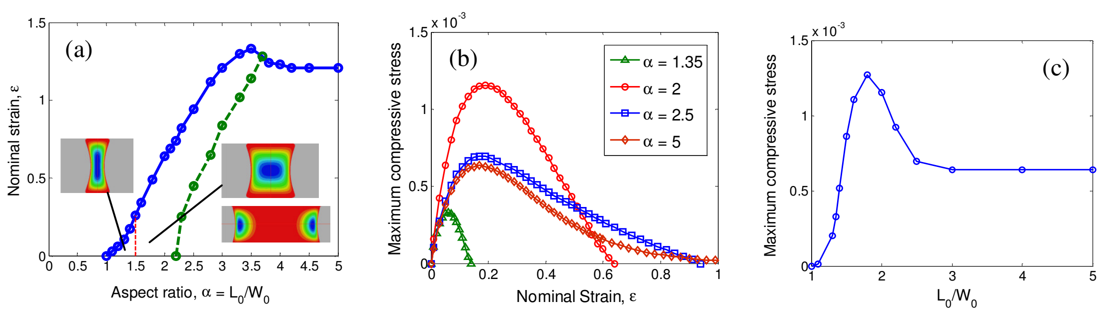

Figure 2: (a) A phase diagram for stretch-induced compressive stress patterns. Insets show three distribution patterns of the compressive transverse stress (σ_y). (b) Variation of the maximum compressive stress magnitude (normalized by $E = 3\mu$) under increasing stretch; (c) Maximum magnitude of the stretch-induced compressive stress as a function of the aspect ratio.

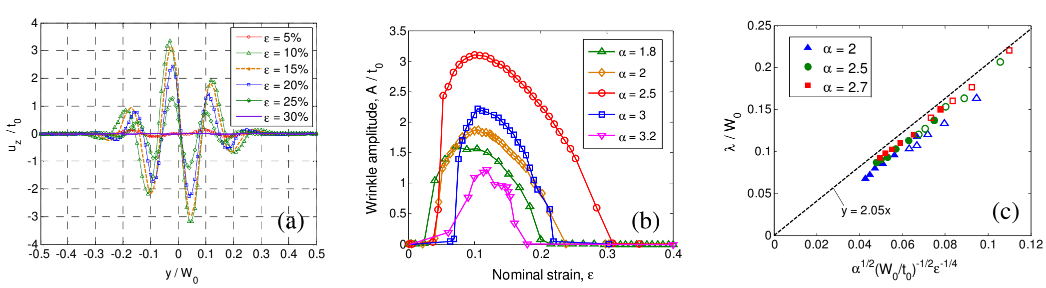

Figure 3. (a) Out-of-plane displacement along the mid-section of the sheet with $\alpha = 2.5$ and $W_0/t_0 = 1000$ under increasing nominal strain. (b) Wrinkle amplitude as a function of the nominal strain. (c) Normalized wrinkle wavelength, in comparison with the prediction by a scaling analysis (dashed line).