A NEW STRAIN-SPACE MICROPLANE PLASTIC-DAMAGE MODEL FOR CONCRETE IN COMPRESSION

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Summary Microplane theory is a class of general approach to establish three-dimensional constitutive relations of engineering materials from the simpler one- or two- dimensional behavior on generic planes with predefined spatial orientations. Though microplane theory successfully applied to a lot of engineering materials, it is still challenging to reconcile the conflict between the thermodynamic consistency and the numerical capability, e.g. in reflecting the deviatoric-induced dilatancy typical in geomaterial. In this work, we present a new strain-space microplane plastic-damage model for concrete in compression, with (major) symmetric material stiffness so that the reported thermodynamical inconsistency can be eliminated. Moreover, as the Reynolds effect is explicitly considered on each microplane under compression by using a non-associated evolution law to model the tangential behavior, the deviatoric-induced dilatancy can be well captured. An additional advantage is that the hysteretic loops during unloading/reloading cycles are also satisfactorily reproduced. Several typical numerical simulations are presented to validate the above conclusions.

GENERAL FORMULATION

According to the macroscopic strain-space plasticity model [1], the stress tensor $\boldsymbol{\sigma}$ and its rate $\dot{\boldsymbol{\sigma}}$ are expressed as

$$\boldsymbol{\sigma} = \mathbb{E} : \boldsymbol{\epsilon} - \boldsymbol{\sigma}^{\mathrm{p}}, \qquad \dot{\boldsymbol{\sigma}} = \mathbb{E} : \dot{\boldsymbol{\epsilon}} - \dot{\boldsymbol{\sigma}}^{\mathrm{i}} \quad \text{with} \quad \dot{\boldsymbol{\sigma}}^{\mathrm{i}} = \dot{\boldsymbol{\sigma}}^{\mathrm{p}} - \dot{\mathbb{E}} : \boldsymbol{\epsilon} \tag{1}$$

for the inelastic strain rate $\dot{\boldsymbol{\sigma}}^{\mathrm{i}}$ contributed from the degradation of the material stiffness $\mathbb{E}$ and from the plastic stress $\boldsymbol{\sigma}^{\mathrm{p}}$. Within the framework of microplane theory, the microplane normal, volumetric, deviatoric and tangential strains, $\epsilon_{\mathrm{N}}, \epsilon_{\mathrm{V}}, \epsilon_{\mathrm{D}}$ and $\boldsymbol{\epsilon}_{\mathrm{T}}$, are resolved as the projections of the macroscopic strain tensor $\boldsymbol{\epsilon}$, i.e.

$$\epsilon_{\mathrm{N}} = \boldsymbol{N} : \boldsymbol{\epsilon}, \qquad \epsilon_{\mathrm{V}} = \boldsymbol{V} : \boldsymbol{\epsilon}, \qquad \epsilon_{\mathrm{D}} = \boldsymbol{D} : \boldsymbol{\epsilon}, \qquad \boldsymbol{\epsilon}_{\mathrm{T}} = \boldsymbol{T} \cdot \boldsymbol{\epsilon} \tag{2}$$

for the corresponding projection operators $\boldsymbol{N}, \boldsymbol{V}, \boldsymbol{D}$ and $\boldsymbol{T}$ in terms of the unit normal vector $\boldsymbol{n}$ of the microplane

$$\boldsymbol{N} = \boldsymbol{n} \otimes \boldsymbol{n}, \qquad \boldsymbol{V} = \frac{1}{3}\boldsymbol{I}, \qquad \boldsymbol{D} = \boldsymbol{n} \otimes \boldsymbol{n} - \frac{1}{3}\boldsymbol{I}, \qquad \boldsymbol{T} = \mathbb{I} \cdot \boldsymbol{n} - \boldsymbol{n} \otimes \boldsymbol{n} \otimes \boldsymbol{n} \tag{3}$$

where $\boldsymbol{I}$ and $\mathbb{I}$ denote the second-order and symmetric fourth-order identity tensors, respectively. It is assumed that the microplane inelastic behavior is characterized by a set of strain-space plastic/damage surfaces

$$f_{\mathrm{V}} = \left|\epsilon_{\mathrm{V}}\right| - \alpha_{\mathrm{V}} \le 0, \qquad f_{\mathrm{D}} = \left|\epsilon_{\mathrm{D}}\right| - \alpha_{\mathrm{D}} \le 0, \qquad f_{\mathrm{T}} = \left|\boldsymbol{\epsilon}_{\mathrm{T}}\right| - \alpha_{\mathrm{T}} \le 0 \tag{4}$$

Accordingly, the inelastic stress rate $\dot{\boldsymbol{\sigma}}^{\mathrm{i}}$ can be determined as an integral relation over a unit hemi-spherical surface Ω, i.e.

$$\dot{\boldsymbol{\sigma}}^{\mathrm{i}} = \frac{3}{2\pi} \int_{\Omega} \left(\dot{\lambda}_{\mathrm{V}} \frac{\partial g_{\mathrm{V}}}{\partial \boldsymbol{\epsilon}} + \dot{\lambda}_{\mathrm{D}} \frac{\partial g_{\mathrm{D}}}{\partial \boldsymbol{\epsilon}} + \dot{\lambda}_{\mathrm{T}} \frac{\partial g_{\mathrm{T}}}{\partial \boldsymbol{\epsilon}} \right) \mathrm{d}\Omega \tag{5}$$

for another set of strain-space potential functions $g_{\mathrm{V}}, g_{\mathrm{D}}$ and g_{T} and the Lagrangian multipliers $\dot{\lambda}_{\mathrm{V}} \ge 0$, $\dot{\lambda}_{\mathrm{D}} \ge 0$ and $\dot{\lambda}_{\mathrm{T}} \ge 0$ constrained by the classical Karush-Kuhn-Tucker loading/unloading conditions. Either associated or non-associated evolution laws can the adopted as needed. Similarly, the material stiffness $\mathbb{E}$ is also expressed in an integral form [2]

$$\mathbb{E} = \frac{3}{2\pi} \int_{\Omega} \left(\phi_{\mathrm{V}} E_{\mathrm{V}}^{0} \boldsymbol{V} \otimes \boldsymbol{V} + \phi_{\mathrm{D}} E_{\mathrm{D}}^{0} \boldsymbol{D} \otimes \boldsymbol{D} + \phi_{\mathrm{T}} E_{\mathrm{T}}^{0} \boldsymbol{T}^{\mathrm{T}} \cdot \boldsymbol{T} \right) \mathrm{d}\Omega \tag{6}$$

where the integrity variables $1 \ge \phi_{\mathrm{V}}(\epsilon_{\mathrm{V}}), \phi_{\mathrm{D}}(\epsilon_{\mathrm{D}}), \phi_{\mathrm{T}}(\boldsymbol{\epsilon}_{\mathrm{T}}) \ge 0$, respectively, characterize the degradation of the microplane volumetric, deviatoric and tangential elastic moduli $E_{\mathrm{V}}^{0}, E_{\mathrm{D}}^{0}$ and E_{T}^{0} (see [3] for the optimal expressions). We note the major symmetry possessed by the material stiffness $\mathbb{E}$ which is necessary to guarantee the existence of a well-defined free energy function.

APPLICATION TO CONCRETE IN COMPRESSION

The internal compressive-friction on a rough weak plane plays an important role on the nonlinear behavior of concrete like geomaterial. It is exactly this mechanism that is responsible for the shear-induced dilatancy and leads to non-associated evolution laws in dominant compression. In this work, we assume that the microplane volumetric and deviatoric evolution laws are associated, whereas a non-associative one is adopted for the tangential behavior, i.e.

$$g_{\mathrm{V}} = f_{\mathrm{V}}, \qquad g_{\mathrm{D}} = f_{\mathrm{D}}, \qquad g_{\mathrm{T}} = \left|\boldsymbol{\epsilon}_{\mathrm{T}}\right| + \mu H\big(-\epsilon_{\mathrm{N}}\big) \cdot \epsilon_{\mathrm{N}} - \alpha_{\mathrm{T}} \tag{7}$$

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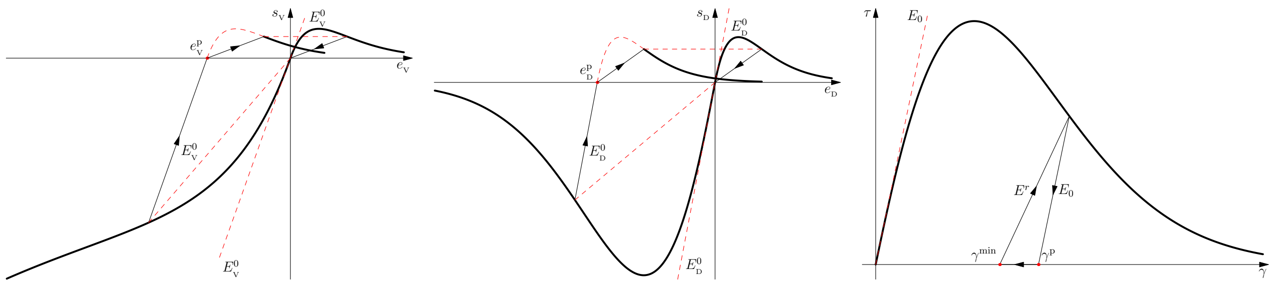
where μ is a model parameter reflecting the shear-induced dilatancy ($\mu = 0.5$ is taken in the numerical simulations), and Heaviside function $H(\cdot)$ is introduced to guarantee that the deviatoric-induced dilatancy is only activated on the microplane under compression. Noticing the kinematic constraint (2) and the inelastic stress rate (5), we have

$$\dot{\boldsymbol{\sigma}}^{\mathrm{i}} = \frac{3}{2\pi} \int_{\Omega} \Big(\boldsymbol{V} \dot{\sigma}_{\mathrm{V}}^{\mathrm{i}} + \boldsymbol{D} \dot{\sigma}_{\mathrm{D}}^{\mathrm{i}} + \boldsymbol{T}^{\mathrm{T}} \cdot \dot{\boldsymbol{\sigma}}_{\mathrm{T}}^{\mathrm{i}} + \boldsymbol{N} \dot{\sigma}_{\mathrm{N}}^{\mathrm{i}} \Big) \, \mathrm{d}\Omega \tag{8}$$

for the microplane components expressed as

$$\dot{\sigma}_{\mathrm{V}}^{\mathrm{i}} = \dot{\lambda}_{\mathrm{V}} \operatorname{sign}(\epsilon_{\mathrm{V}}), \qquad \dot{\sigma}_{\mathrm{D}}^{\mathrm{i}} = \dot{\lambda}_{\mathrm{D}} \operatorname{sign}(\epsilon_{\mathrm{D}}), \qquad \dot{\boldsymbol{\sigma}}_{\mathrm{T}}^{\mathrm{i}} = \dot{\lambda}_{\mathrm{T}} \boldsymbol{1}_{\mathrm{T}}, \qquad \dot{\sigma}_{\mathrm{N}}^{\mathrm{i}} = \mu H\big(-\epsilon_{\mathrm{N}}\big) \cdot \dot{\lambda}_{\mathrm{T}} \tag{9}$$

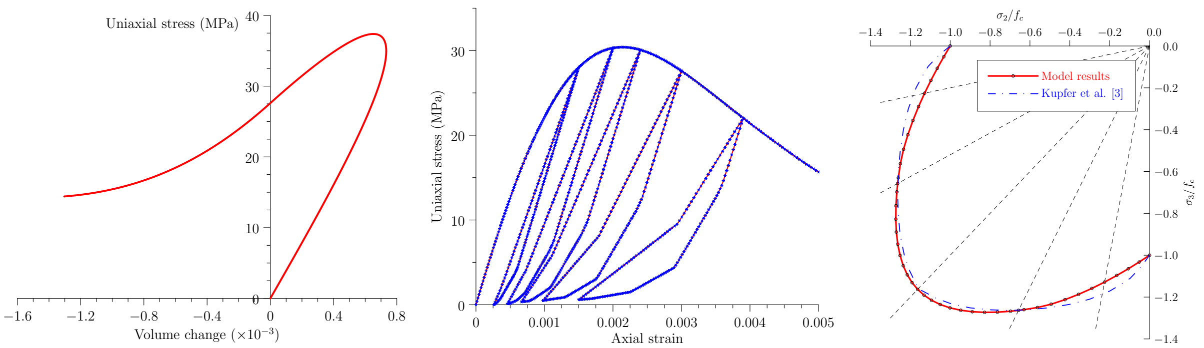
Upon loading the Lagrangian multipliers, $\dot{\lambda}_{\mathrm{V}} > 0$, $\dot{\lambda}_{\mathrm{D}} > 0$ and $\dot{\lambda}_{\mathrm{T}} > 0$, can be determined from the consistency conditions. The details are omitted here. As we can see, the inelastic tangential stress on a microplane in compression will induce some inelastic normal stresses, i.e. the well-known Reynolds effect. This property is necessary for the modeling of compressive dilatancy typical in concrete like engineering materials.



(a) Volumetric stress-strain relationship (b) Normal deviatoric stress-strain relation (c) Tangential stress-strain relation

Figure 1. Stress-strain relations at the microplane level.

Using the microplane stress-strain relations (similar to those proposed in [3]) shown in Fig.1, we simulated the nonlinear behavior of concrete in uniaxial compression, cyclic compression and biaxial compression. The results are given in Fig.2.



(a) Dilatancy in uniaxial compression (b) Cyclic compression (c) Biaxial strength envelop

Figure 2. Representative numerical results

CONCLUSIONS

The proposed microplane plastic-damage model is not only thermodynamically consistent, but also is capable of intrinsically capturing the Reynolds effect. The presented numerical results demonstrate that the typical nonlinear behavior of concrete in dominant compression, e.g. the deviatoric-induced dilatancy, the hysteretic loops during the unloading/reloading cycles, and the strength enhancement under lateral confinement, etc., can be satisfactorily reflected.

References

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