ON CONSTITUTIVE EQUATIONS FOR PROCESSES OF SEVERE PLASTIC DEFORMATION

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Summary Processes of severe plastic deformation are used to significantly change the structure of material to improve its mechanical and physical properties. Usually, material models adopted to describe such deformation processes do not differ from the models used in conventional plasticity. However, because of unique technological features of the processes of severe plastic deformation, it is natural to expect that mathematical models for these processes should incorporate some of these features. The present paper is a preliminary attempt to formulate such a model. The specific focus is on the effect of the rotation the principal axes of the stress tensor (or strain rate tensor) relative to material on the evolution of grain size .

SEVERE PLASTIC DEFORMATION AND FRICTION

In traditional processes of bulk severe plastic deformation the distribution of strain and grain size is nearly uniform in the final product [1]. A unique feature of all these processes is that the motion of tool and constraints imposed tend to increase the rotation of the principal axes of the stress tensor relative to material. Therefore, it is natural to hypothesize that a measure of this rotation (or the rate of this rotation) plays an important role in the formation of material structure during processes of severe plastic deformation. An indirect confirmation of this hypothesis follows from microstructure observations in the vicinity of surfaces where high friction stresses occur in traditional metal forming processes. In particular, the distribution of grain size after extrusion of round rods of a magnesium alloy through conical dies with different angles is shown in Figs. 1 to 3 [2]. It is seen from these figures that there is a narrow layer of fine grains in the vicinity of the friction surface. On the other hand, it is known that the velocity component tangent to maximum friction surfaces (surfaces where the friction stress is equal to the shear yield stress) follows the rule [3]

$$u_\tau = U_0 + U_1\sqrt{s} + o\left(\sqrt{s}\right), \quad s \to 0 \tag{1}$$

where s is the distance to the friction surface and U_0 and U_1 are independent of s. It is evident from (1) that some components of the spin tensor approach infinity near the friction surface. On the other hand, the orientation of the principal stress axes is fixed in space. This theoretical result combined with the aforementioned hypothesis is in qualitative agreement with the experiment (Figs.1 to 3). Rigid perfectly plastic models have been used in [3]. However, it is known that equation (1) is valid for other rigid plastic models as well. A review can be found in [4].

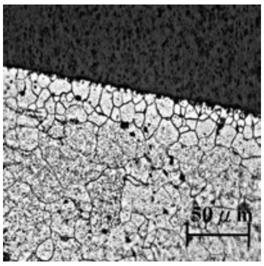

Fig.1 Structure of material near the friction surface (die angle = 5^0)

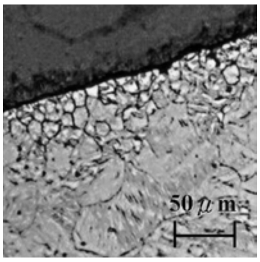

Fig.2 Structure of material near the friction surface (die angle = 10^0)

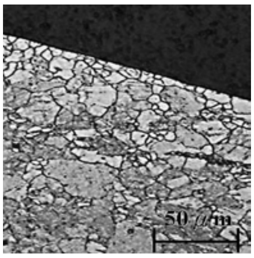

Fig.3 Structure of material near the friction surface (die angle = 15^0)

SEVERE PLASTIC DEFORMATION AND IDEAL FLOWS

Ideal plastic flows are those in which one family of material lines is perpetually tangent to principal strain rate and principal stress vectors [5]. In the case of plane strain and axisymmetric processes, all three principal stress vectors are fixed in material. Therefore, in contrast to processes of severe plastic deformation, there is no rotation of the principal stress axes relative to material. On the other hand, multi-step ideal flow processes, such as extrusion through several dies of special shape, can be used to deform material to a very high level of strain comparable with that in traditional processes of severe plastic deformation. Therefore, such processes are ideal for revealing a possible effect of the rotation

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of the principal stress axes relative to material on the formation of its structure and thus for verifying the hypothesis formulated. Further experimental studies will need to be undertaken in this direction.

CONSTITUTIVE EQUATIONS

At this stage of research the constitutive equations proposed are the system of constitutive equations of conventional plasticity theory supplemented by an evolution equation for grain size. For reasons of space, consideration is restricted to plane strain deformation. In order to introduce a measure of the rotation of the principal stress axes relative to material, it is necessary to include the material spin tensor in the formulation. The only non-zero component of this tensor relative to an arbitrary orthogonal coordinate system (λ, β) is given by

$$\omega_{\lambda\beta} = \frac{1}{2H_\lambda H_\beta}\left[\frac{\partial\left(H_\beta u_\beta\right)}{\partial\lambda} - \frac{\partial\left(H_\lambda u_\lambda\right)}{\partial\beta}\right] \tag{2}$$

where H_β and H_λ are the scale factors of the coordinate curves and u_β and u_λ are the velocity components. Let ψ be the angle between the $\lambda-$direction and the principal axis corresponding to the major principal stress σ_1 at a generic point, measured from the coordinate line anti-clockwise. The convective derivative of ψ is defined by

$$\dot{\psi} = \frac{\partial\psi}{\partial t} + \frac{u_\lambda}{H_\lambda}\frac{\partial\psi}{\partial\lambda} + \frac{u_\beta}{H_\beta}\frac{\partial\psi}{\partial\beta} \tag{3}$$

where $\partial\psi/\partial t$ is the derivative at a point which is fixed relative to the $\lambda\beta-$coordinates. Then, a possible measure of the rotation of the principal stress axes is

$$\Omega = \left(\omega_{\lambda\beta} - \dot{\psi}\right)/\xi_{eq} \tag{4}$$

where the equivalent strain rate ξ_{eq} has been introduced to represent Ω in dimensionless form. The evolution equation for the grain size *R* is proposed in the following form

$$dR/d\varepsilon_{eq} = -R_s\left(R/R_s - 1\right)^\eta \Phi\left(R/R_s, \varepsilon_{eq}, \Omega\right) \tag{5}$$

where R_s is the minimum possible grain size (the existence of such a value is confirmed by many experiments), $\eta \geq 1$, the equivalent strain ε_{eq} is defined by the equation $d\varepsilon_{eq}/dt = \xi_{eq}$, and Φ is a function of its arguments, even in Ω.

CONCLUSIONS

Based on a review of available experimental data and theoretical results concerning the state of stress and strain rate in processes in which a fine grain structure is obtained, it has been proposed to include a measure of the rotation of the principal stress axes in the formulation of constitutive equations for processes of severe plastic deformation. A specific equation has been proposed for the evolution of grain size. An experimental program has been worked out to verify the new approach to the formulation of constitutive equations for processes of severe plastic deformation. Equation (5) can be used in analysis of conventional processes of severe plastic deformation with no difficulty. Such analysis could eventually allow one to specify the function Φ for given material.

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