A MODIFIED FRACTIONAL ZENER MODEL FOR DYNAMIC MECHANICAL PROPERTIES OF CB-FILLED RUBBER

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Summary Frequency-sweep tests at various temperatures of carbon black(CB) filled rubber have been carried out, master curve of the loss factor is obtained. Based on the experimental results, the fractional Zener model is verified to be unsuitable to reproduce the dynamical viscoelastic behaviours of the CB-filled rubber, and a modified fractional Zener model with two spring-pots is developed. The applicability of the proposed model is proved by the experimental data.

INTRODUCTION

Rubber material used in tires, seals and vibration dampers and many other components is well known to us. However, since rubbers show markedly viscoelastic properties, their modulus depends on time, temperature, which is important for the warranties of one or two decade on the rubber components. Fractional calculus-based constitutive models are developed to represent the material's viscoelasticity in recent years, and most of them can reproduce the viscoelastic behavior of polymer materials over a wider range of temperature and frequency accurately. In this work, we modify the fractional Zener model by introducing an additional element in order to obtain the asymmetric response for the loss factor and the Cole-Cole diagram of the filled rubbers. The applicability of the modified fractional Zener model is checked then by a comparison with experimental results.

DYNAMIC VISCOELASTICITY AND MASTER CURVES

The test samples with dimensions 35x7x2 mm^3 were cut from a carbon black filled rubber sheet. Frequency-sweep tests at various temperatures have been carried out at different temperatures varying from -70 °C to 50 °C. All tests were performed on a Gabo Eplexor 500N under a prestrain of 0.4% and superimposed dynamic strain amplitude of 0.1%.
According to the frequency-temperature superposition, the master curve of loss factor versus frequency for reference temperature 30°C and the Cole-Cole diagram are plotted in Fig.1 and Fig.2, they are found to be asymmetric.

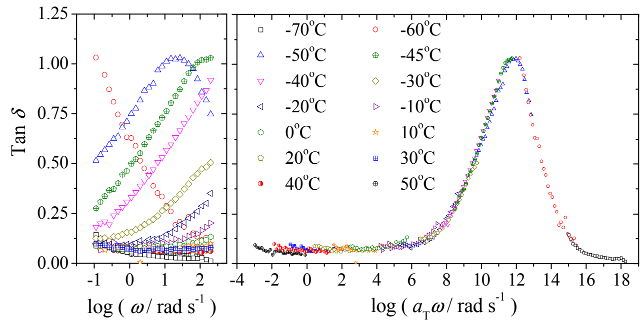

Fig. 1. Loss factors of CB filled rubber at various temperatures (left) and the master curve for 30°C (right)

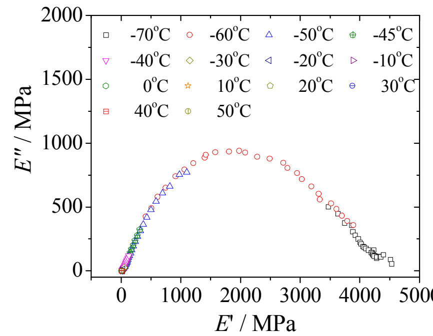

Fig. 2. Cole-Cole diagram of CB filled rubber

FRACTIONAL CALCULUS MODEL

Fractional Zener model

Fig.3 shows the fractional Zener model. The dynamic moduli and loss factor of the fractional Zener model are:

$$E'(\omega) = E_0 + \frac{E_1\tau^{2\alpha}\omega^{2\alpha} + E_1\tau^{\alpha}\omega^{\alpha}\cos(\alpha\pi/2)}{1+\tau^{2\alpha}\omega^{2\alpha}+2\tau^{\alpha}\omega^{\alpha}\cos(\alpha\pi/2)} \quad (1)$$

$$E''(\omega) = \frac{E_1\tau^{\alpha}\omega^{\alpha}\sin(\alpha\pi/2)}{1+\tau^{2\alpha}\omega^{2\alpha}+2\tau^{\alpha}\omega^{\alpha}\cos(\alpha\pi/2)} \quad (2)$$

$$\tan\delta = \frac{E''}{E'} \quad (3)$$

Fig.4 and Fig.5 show the prediction of loss factor versus frequency for different values of α parameter, with the assumption of E_0=E_1. From these diagrams, we can find that the loss factor and Cole-Cole plots of the fractional model are symmetric, which are not consistent with the observation in dynamic mechanical tests of the filled rubber as shown in Fig.1 and Fig.2.

Modified fractional Zener model

To capture the real viscoelastical response for the CB filled rubber and other polymers, we introduce an additional

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spring pot into the classic fractional Zener model, and a modified fractional Zener model is proposed as shown in Fig.6. The dynamic mechanical properties of the modified fractional Zener model are given as below:

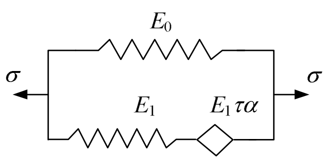

Fig.3. Fractional Zener model

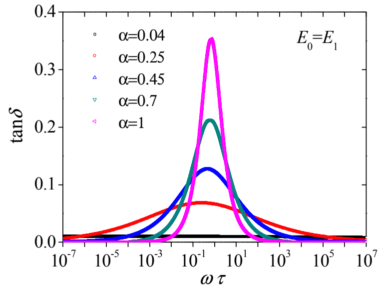

Fig.4. Loss factors versus frequency for fractional Zener model

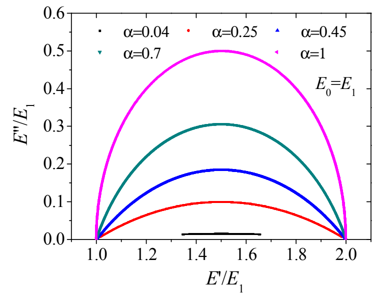

Fig.5. Cole-Cole diagrams for fractional Zener model

$$E' = E_0 + \frac{E_1 AB\left[AB + B\cos(\beta\pi/2) + B\cos(\alpha\pi/2)\right]}{A^2B^2 + 2AB\cos(\beta\pi/2)\cos(\alpha\pi/2) + 2AB^2\cos(\beta\pi/2) + 2A^2B\cos(\alpha\pi/2) + 2AB\sin(\beta\pi/2)\sin(\alpha\pi/2) + A^2 + B^2} \tag{5}$$

$$E'' = \frac{E_1 AB\left[B\sin(\beta\pi/2) + A\sin(\alpha\pi/2)\right]}{A^2B^2 + 2AB\cos(\beta\pi/2)\cos(\alpha\pi/2) + 2AB^2\cos(\beta\pi/2) + 2A^2B\cos(\alpha\pi/2) + 2AB\sin(\beta\pi/2)\sin(\alpha\pi/2) + A^2 + B^2} \tag{6}$$

$$\tan\delta = \frac{E''}{E'} \quad \text{where} \quad A = \omega^\beta\tau_1^\beta,\ B = \omega^\alpha\tau_2^\alpha,\ 0<\alpha\le\beta\le 1. \tag{7}$$

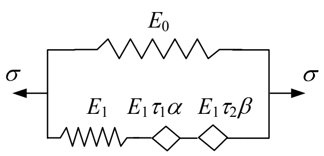

Fig.6. A modified fractional Zener model

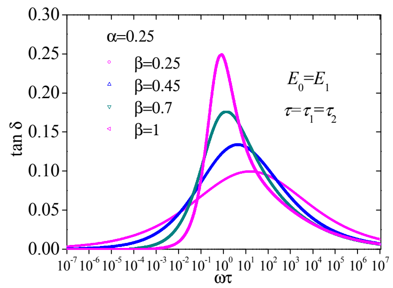

Fig.7 Loss factors versus frequency for modified fractional Zener model

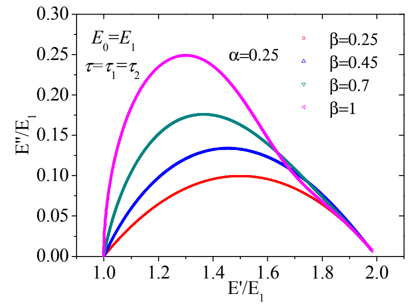

Fig.8 Cole-Cole diagram of the modified fractional Zener model

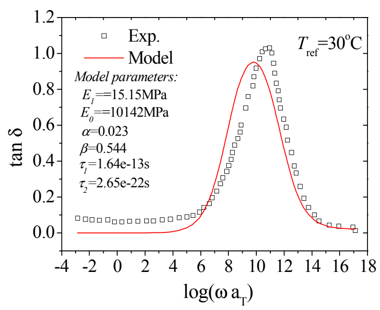

Fig.9 loss factor of CB-filled rubber:test vs. model

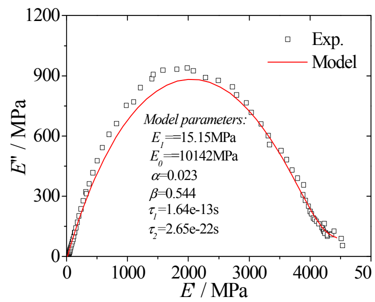

Fig.10 Cole-Cole diagram of CB-filled rubber: test vs. model

Loss factors and Cole-Cole plots of the modified model are shown in Fig.7 and Fig.8 for a constant parameter α=0.25 and different parameter β, with the assumption of $E_0=E_1$, $\tau_1=\tau_2$. It is obvious that by introducing an additional spring pot into the classic fractional Zener model, the model can reproduce the asymmetric response of CB filler rubber in loss factors and Cole-Cole plots.

To verify the applicability of the modified model, experimental results of the filled rubber shown in Fig.1 and Fig.2 are simulated as shown in Fig.9 and Fig.10. We see very good agreement for higher frequency, but at low frequency, the modified model is unsuitable.

CONCLUSIONS

Comparing the experimental results and theoretical analyses of the fractional Zener model, the model is pointed out to be inaccurately in describing the asymmetric property of real materials, and a new model by adding a spring-pot to the fractional Zener model is developed. An agreement between experiments and model has been achieved at a wide range of working conditions.

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