STEADY GRANULAR FLOW: LOCAL AND NONLOCAL APPROACHES

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Summary A general, three-dimensional law to predict granular flow in arbitrary geometries has remained an elusive challenge in engineering. Recently, an elasto-plastic continuum model has shown the ability to approximate steady flow and stress profiles in multiple inhomogeneous flow environments. However, the model does not capture some phenomena observed in the slow, creeping flow regime. As flow-rate decreases, granular stresses are observed to become largely rate-independent and a dominating length-scale emerges in the mechanics. This talk accounts for these effects using the notion of nonlocal fluidity, which has proven successful in treating nonlocal effects in emulsions. We augment the usual granular fluidity law with a diffusive second-order term scaled by the particle size that spreads flowing zones accordingly. Below the yield stress, the local contribution vanishes and the fluidity becomes rate-independent, as we require. We implement the modified law in multiple geometries and validate its predictions for velocity, shear-rate, and stress against discrete particle simulations.

GOALS AND BACKGROUND

Dense granular materials such gravel and sand compose a family of heterogeneous media with a number of modeling challenges. While discrete element approaches have been useful and have had success in reproducing many experimental results, a feasible and broadly applicable continuum law for granular flow is much desired, that can predict stress and flow fields in arbitrary geometries. We are interested here in *steady-state flow* behavior, though granular transients in strength and porosity are themselves a major challenge.

A local rheology has emerged over the last decade that appears to do well representing the rheology of homogeneously flowing materials. The plastic deformation takes the form of a viscoplastic rheology similar to a Bingham or over-stress model. In 2D simple shear, it can be written as $\mu = \mu(I)$ in accord with dimensional arguments, where $\mu = \tau/P$ for shear stress τ and normal pressure P, and I is the inertial number $\dot{\gamma}\sqrt{m/P}$ for particle mass m. Inverted, linearized, and written as a strain-rate law it reads [1]

$$\dot{\gamma}_{loc}(\mu) = \sqrt{P/m}\, H(\mu - \mu_s)\, (\mu - \mu_s)/b \tag{1}$$

for constant b and static yield criterion μ_s, and Heaviside function H.

This basic relation can be extended to a three-dimensional from by presuming codirectionality of the strain-rate and the Cauchy stress [2], i.e. isotropic extension. The result has proven effective in certain geometries and was recently appended with a nonlinear elastic mechanism to represent stresses in material below yield [3]. In this work, evidence of a roughly five-particle-diamater instantaneous Representative Volume Element was provided for a number of common flows, validating the usage of a continuum description for flowing granular matter as well as the coaxial presumption.

NONLOCAL EFFECTS

While the local flow relation is highly general and can be implemented somewhat straightforwardly using finite-element analysis, there are a number of circumstances where granular flows do not appear to obey this relation due to finite-size effects. Because grains are commonly macro-sized, and flow environments can often be less than thousands of grains, these effects are not difficult to come across.

In steady flow, the major nonlocal effect is that material seems to flow (albeit slowly) in regions where $\mu < \mu_s$ as long as a strain-rate gradient is present. While the local model predicts a sharp flow/no-flow interface to occur on the locus $\mu = \mu_s$, instead an exponentially decaying flow profile is generally observed that dies off gradually into the region below μ_s. The width of this "blur" scales with the particle size, indicating the importance of the intrinsic length-scale.

Furthermore, surprisingly, flow in quasi-static zones becomes almost completely rate-independent in steady-state. This fact runs counter to the $\mu = \mu(I)$ notion of the local law, since many possible I values can correspond to the same μ value in regions below μ_s. This type of behavior can also be deemed a nonlocal effect because, rather than usual rate-independent J_2 plasticity, a range of subyield steady-state strength values is observed in the steadily flowing material, which cannot be explained locally.

A SIMPLE NONLOCAL APPROACH

We have found a gradient-based correction to the local rheology that appears to capture well both the size-effect in decaying granular flow profiles, and the nonstandard rate-independence in quasi-static regions. Our new model is motivated by successful previous work in the field of emulsions , which utilizes the notion of a nonlocal fluidity [4]. We define the

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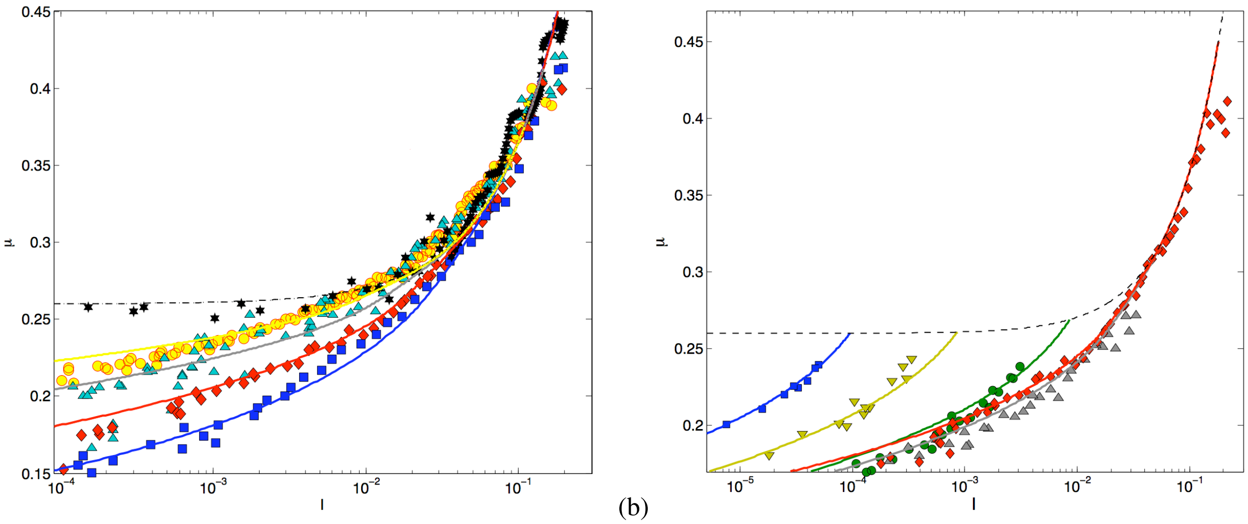

(a) (b)

Figure 1. μ vs. I locus in steady annular shear: (a) Nonlocal model (solid lines) compared to Discrete Element Method (DEM) data for $\tilde{V}_{wall} = 2.5$ and $\tilde{R} = 25(\square),\ 50(\Diamond),\ 100(\triangle),\ 200(\circ)$. (b) Comparisons for fixed $\tilde{R} = 50$ and $\tilde{V}_{wall} = 0.00025(\square),\ 0.0025(\bigtriangledown),\ 0.025(\circ),\ 0.25(\triangle),\ 2.5(\Diamond)$. The local rheology (dotted line) is indicated, to which the nonlocal model approaches only in uniform stress environments (e.g. planar shear, DEM data ($\star$) of [5]).

granular fluidity by

$$g = \dot{\gamma}/\mu. \tag{2}$$

Let $\xi = \xi(\mu)$ be the length for cooperative particle rearrangement (scaled by the particle diameter). We propose that the steady fluidity obeys

$$g - g_{loc} = \xi^2 \nabla^2 g \tag{3}$$

for $g_{loc} = \dot{\gamma}_{loc}/\mu$. Hence, for a simple, known stress field, one solves the differential relation above for g and then substitutes the result into the prior equation to determine the flow-rate field. Note that in the absence of gradients, the relation reduce to the local law. When the local law has no contribution (i.e. $g_{loc} = 0,\ \mu < \mu_s$), the differential relation becomes a linear equation whose solutions can always be scaled by a constant — this gives precisely the rate-independent effect observed. And due to the Laplacian term, flow naturally diffuses out near μ_s with a decay determined by ξ, instead of a sharp flow/no-flow interface.

We have calibrated and validated this model in several geoemtries. An example of its abilities can be seen in the figure above, which compares the model to steady discrete particle simulations in the annular couette geometry. In the left figure, different data sets correspond to different values of the ratio of geometry radius to the particle size, $\tilde{R} = R/d$, and the right figure shows data for different values of normalized wall speed $\tilde{V}_{wall}$. The dotted line indicates the local law, which predicts the same μ vs I relation in all cases. It is clear that the nonlocal model does a far superior job in slow flow, correctly predicting the split-off from a single rheological curve.

The same model, with same parameters, was tested against discrete simulations of the same composition in two completely different geometries (e.g. vertical chute flow, shear under gravity) with similar success. It is shown to give flow predictions that match the steady velocity of the discrete simulation for over three orders of magnitude in relative speed.

CONCLUSIONS

A nonlocal enhancement to the standard granular rheology was tested in multiple geometries. The enhanced model was shown able to describe the various aspects of flow nonlocality observed in steady granular deformation. It is also able to quantitatively match the flow profiles in multiple geometries while varying multiple geometric parameters.

References

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