CORRECTION OF SHEAR CREEP COMPLIANCE DETERMINED BY CONICAL INDENTATION

Guangjian Peng*, Taihua Zhang*,a) & Yihui Feng*

* *State Key Laboratory of Nonlinear Mechanics (LNM), Institute of Mechanics, Chinese Academy of Sciences, Beijing 100190, China*

Summary It is found that the shear creep compliance determined by conical indentation is systematically smaller than that determined by uniaxial tension which is equal to the theoretical result. Four potential influencing factors on the systematic error are discussed. A correction factor which relates to the Poisson's ratio and relaxation factor of sample and the half-angle of conical indenter is given to correct the shear creep compliance determined by conical indentation.

Instrumented indentation is an efficient and convenient tool for probing time-dependent behaviors of polymers and biomaterials. Based on the viscoelastic indentation solution presented by Lee and Radok[1], several methods[2-4] for determination of shear creep compliance from indentation measurements have been proposed. Specifically, for *Step-Hold* load, a step load is suddenly exerted on sample and holds for some time, the indentation load can be expressed as $F(t)=F_0H(t)$, where $H(t)$ is the Heaviside unit step function. The formula for computing shear creep compliance is[2,4]

$$J(t)=\frac{4\tan\alpha}{\pi(1-\nu)F_0}h^2(t), \tag{1}$$

where α is the half-angle of conical indenter, ν is the Poisson's ratio of sample.

For *Ramp* load, a load increases from zero to peak load at a constant loading rate, the loading condition can be written as $F(t)=V_\mathrm{F}t$, with V_F being the loading rate. And the formula for computation of shear creep compliance is[2,4]

$$J(t)=\frac{4\tan\alpha}{\pi(1-\nu)V_\mathrm{F}}\frac{dh^2(t)}{dt}. \tag{2}$$

However, as the measured data is discrete, the computation of the derivative dh^2/dt will induce large error in the creep function. A fitting method is adopted to determine the shear creep compliance. According to the generalized Kelvin model, the shear creep function can be written in the form of Prony series[2,3]

$$J(t)=J_\infty-\sum_{i=1}^{N}J_ie^{-t/\tau_i}, \tag{3}$$

where J_∞ is the long-term creep compliance; J_i and τ_i are compliance constant and retardation time, respectively; N is a positive integer. Substituting Eq. (3) into Eq. (2), and then integrating it, we get

$$h^2(t)=\frac{\pi(1-\nu)V_\mathrm{F}}{4\tan\alpha}\left[J_\infty t-\sum_{i=1}^{N}J_i\tau_i\left(1-e^{-t/\tau_i}\right)\right]. \tag{4}$$

Fitting Eq. (4) with the measured depth-time data by the least squares method, we can get a set of best-fit parameters J_∞, J_i and τ_i. Then the shear creep compliance can be determined by substituting these parameters back into Eq. (3).

Numerical experiments are implemented in ABAQUS to verify the two methods. A frictionless, rigid conical indenter indenting an isotropic linear viscoelastic solid with *Step-Hold* load and *Ramp* load are considered. Uniaxial tensile tests are also simulated. "Three-element solid" (a spring in parallel with a dashpot, the pair in series with a second spring) with time-independent Poisson's ratio is used. And the theoretical result on shear creep compliance can be computed by

$$J(t)=2(1+\nu)\left[\frac{1}{E_0}+\frac{1}{E_1}\left(1-e^{-tE_1/\eta_1}\right)\right], \tag{5}$$

where $1/E_0$ and $1/E_0+1/E_1$ are instantaneous compliance and long-term compliance, respectively; η_1/E_1 indicates the retardation time. Two groups of material parameters are used. For the first group, the instantaneous Young's modulus takes values 0.5, 2.5, 4.5 and 10 GPa; the Poisson's ratio takes values 0.33, 0.38, 0.43 and 0.48; the relaxation factor of shear modulus (equals that of bulk modulus) takes value 0.2; relaxation time is 2 s; and indenter half-angle takes values 62.8°, 65.3°, 67.8° and 70.3°. The combination of these parameters leads to 64 cases. Second group has 48 cases, the instantaneous Young's modulus takes values 0.5, 2.5, 4.5 and 10 GPa; the Poisson's ratio takes values 0.33, 0.38, 0.43 and 0.48; the relaxation factor takes values 0.2, 0.4 and 0.6; relaxation time is 2 s; and indenter half-angle takes value 70.3°. For the *Step-Hold* load, the step load is applied within 0.001s, and then holds for 100s. For the *Ramp* load, the load increases linearly to the peak load in 100s.

The shear creep compliances determined by indentation are compared with the uniaxial tensile result and theoretical result. Part of the results is plotted in Fig. 1, and all the results have the same trend. The uniaxial tensile results consist perfectly with the theoretical results indicates that the viscoelastic model implemented in ABAQUS is correct. But the indentation results are systematically smaller than the theoretical result. The result determined by *Ramp* method is larger than the theoretical result at the very beginning is because fitting Eq. (4) with the measured depth-time data induces fitting error at the beginning. Consequently, in order to find out the influencing factors on the systematic error, only *Step-Hold* method is used. Since the relative error varies with time, as shown in Fig. 3(a), the long-term relative

a) Corresponding author. Email: zhangth@lnm.imech.ac.cn

error (the relative error at 100 s) is used to analyze each influencing factor. Four potential influencing factors, instantaneous Young's modulus, Poisson's ratio, relaxation factor and indenter half-angle are considered.
From Fig. 2, we could find out that the systematic error (a) decreases linearly with $\tan\alpha$, (b) increases linearly with Poisson's ratio, (c) almost does not change with instantaneous Young's modulus, and (d) decreases linearly with relaxation factor. And the correction factor is the reciprocal of "1 + relative error". Fig. 4 shows that the correction factor increases with $\tan\alpha$ with the slope 0.0148, decreases with Poisson's ratio with the slope -0.1320, and increases with relaxation factor with the slope 0.0296. Considering such factors comprehensively, we can get a correction factor $\gamma = 0.0148\tan\alpha - 0.132\nu + 0.0296 g_1 + 1.0379$, where g_1 is the relaxation factor. Since the relaxation factor varies with time during testing, it can approximately be substituted with $1 - J(0)/J(t)$, where $J(0)$ and $J(t)$ are the shear creep compliance before correction. Then the shear creep compliance determined by indentation can be corrected by

$$J_r(t) = \left[0.0148\tan\alpha - 0.132\nu + 0.0296\left(1 - J(0)/J(t)\right) + 1.0379\right] J(t), \tag{6}$$

The relative errors of indentation results range from about 2% to 6% before correction, and decrease to less than 1% after correction (see Fig. 3).

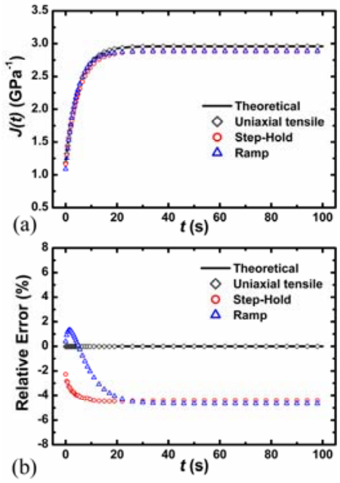

Fig. 1 Indentation results are compared with uniaxial tensile result and theoretical result.

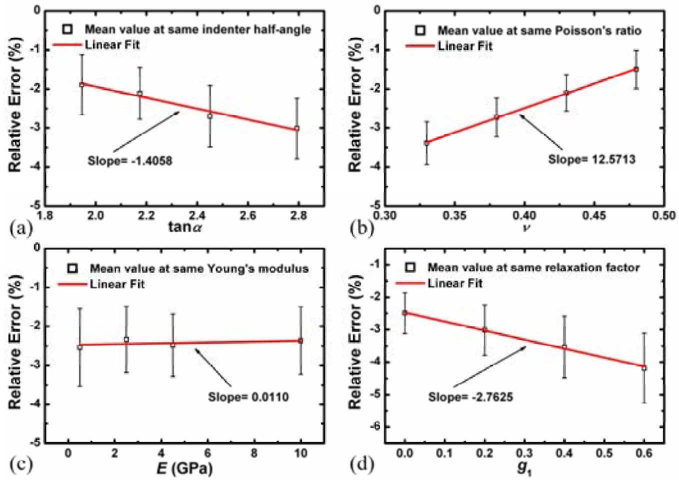

Fig. 2 The influence of (a) indenter half-angle, (b) Poisson's ratio, (c) Young's modulus and (d) relaxation factor on relative error.

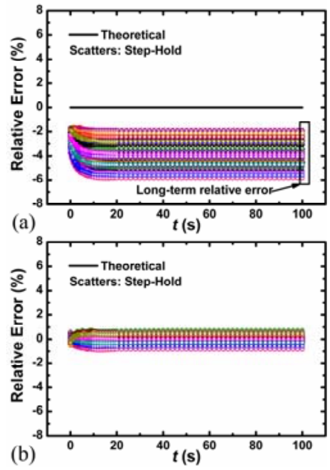

Fig. 3 Relative errors of indentation results: (a) before correction, (b) after correction.

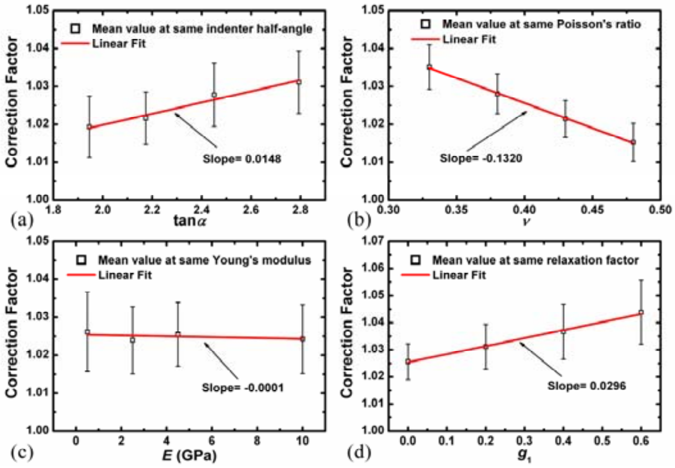

Fig. 4 The influence of (a) indenter half-angle, (b) Poisson's ratio, (c) Young's modulus and (d) relaxation factor on correction factor.

In summary, a large number of numerical experiments are implemented to verify the indentation methods for determination of shear creep compliance. It is found that the indentation result is systematically smaller than the uniaxial tensile result which is equal to the theoretical result. The systematic error is associated with half-angle of conical indenter, Poisson's ratio and relaxation factor of sample. This may help us probe the physical reason for the systematic error. Finally, a correction factor is given to correct the shear creep compliance determined by *Step-Hold* method. And satisfied results are obtained.

References

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