ANALYSIS OF INDENTATION SIZE EFFECTS UNDER STRAIN GRADIENT VISCOPLASTICITY

Suman Guha[*a], Sandeep Sangal[*]& Sumit Basu[**]

[*]*Department of Materials Science and Engineering, Indian Institute of Technology Kanpur, Kanpur, 208016, India*

[**]*Department of Mechanical Engineering, Indian Institute of Technology Kanpur, Kanpur, 208016, India*

Summary Indentation size effects have been observed to exist extensively in several micro/nano-indentation experiments. Different strain gradient plasticity model has been proposed by several researchers to capture the observed size effects. Nix and Gao's gradient plasticity simulations has been found to comply well with the experimental observations mostly at mircon scales. In this article a large deformation finite element simulations have been carried out to analyze indentation of an elastic half space by an wedge shaped indenter based on Fleck-Hutchinson's gradient plasticity formulation. The behavior of the size effects in indentation has been studied based on the varying mechanical properties and the results are found to be agreeing well with the experimental trends.

INTRODUCTION

It has been well established that materials exhibit more resistance to deformation at small scale, which is commonly known as size effect or scale effect. Size effects have been observed to exist predominantly in most of the micro/nano-indentation hardness measurement. In case of most of the microindentation experiments it has been observed that the square of the hardness generally follows a linear trend with the inverse of the indentation depth. Nix and Gao proposed a gradient plasticity theory, based on the concepts of Geometrically Necessary Dislocations (GNDs) which is able to capture the observed scaling effect[3]. The fundamental basis of this theory lies on the Taylors dislocation model for flow stress: $\tau = \alpha\mu b\sqrt{\rho_t}$, where τ is the flow stress in shear and $\rho_t = \rho_s + \rho_g$, where ρ_t is the total dislocation density and ρ_s and ρ_g are the densities of statistically stored dislocations and geometrically necessary dislocations respectively. μ is the shear modulus and b is the Bergers vector. α is an empirical constant. However the observed linear variation in indentation can be predicted only for the indenters of particular geometry[6] (e.g. conical, wedge shaped, Berkovich etc.) and only when the total dislocation density is assumed to be given by the direct sum of ρ_s and ρ_g. It has also been reported that in case of nano-indentation the observed linearity is lost for various materials[4]. Fleck-Hutchinson's (FH) higher order gradient plasticity theory is also capable of capturing the size effect. However the FH model incorporates a relationship which assumes a harmonic mean of ρ_s and ρ_g to derive ρ_t. Accordingly the above mentioned linear variation is not reproduced for indentation simulations, if FH model is used. From various indentation hardness data summarized in [4] it is observed that materials categorized according to their mechanical property exhibit a common trend. It has been identified that the hardness trends vary mainly with the yield strength, the hardening parameter and the characteristic length scale (l_*) of the material. In this work, 2D large deformation finite element simulations have been carried out using FH gradient plasticity theory to observe the effect of the materials yield strength, strain hardening parameter and l_* on the hardness trends for indentation of an elastic half space by an wedge shaped indenter.

FINITE ELEMENT FORMULATION

In order to formulate the finite element equations the virtual work principle for strain gradient material is obtained by assuming the existence of a higher order traction which induces a higher order stress in the material as[2]:

$$\int_V (\sigma_{ij}\delta\epsilon^e_{ij} + Q\delta\epsilon^p + \tau_i\delta\epsilon^p_{,i})dV = \int_S (T_i\delta u_i + t\delta\epsilon^p)dS, \tag{1}$$

where τ_i is the higher order stress work conjugate to the plastic strain gradient $\epsilon^p_{,i}$, Q is the scalar microstress, work conjugate to the equivalent plastic strain ϵ^p and t is the higher order traction. In the absence of strain gradient (1) reduces to the virtual work principle for conventional plasticity and $Q = \sigma_e$, where σ_e is the equivalent stress defined in the conventional manner as $\sigma_e = \sqrt{\sigma'_{ij}\sigma'_{ij}}$ and $\sigma'_{ij} = \sigma_{ij} - \frac{1}{3}\delta_{ij}\sigma_{kk}$. The incremental form of the virtual work principle is derived for large deformation and in an updated lagrangian framework according to [5] using the uniaxial viscoplastic stress-strain law

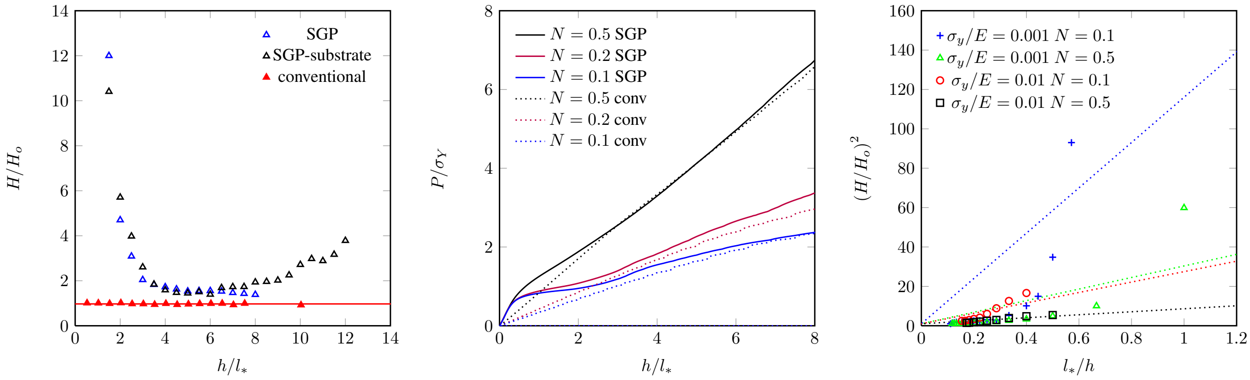
$$\sigma_c = \sigma_0(1 + \frac{E_p}{\epsilon_0})^{1/n}(\dot{E}_p/\dot{\epsilon}_0)^m, \tag{2}$$

where m is the strain rate sensitivity and $\dot{\epsilon}_0$ is the reference strain rate. $\dot{E}_p$ is the effective plastic strain rate defined as

$$\dot{E}_p^2 = \dot{\epsilon}^{p^2} + l_*^2\dot{\epsilon}^p_{,i}\dot{\epsilon}^p_{,i}, \tag{3}$$

where l_* is the characteristic length scale. The present formulation is different from the conventional finite deformation based elasto-plasticity in a manner that both the velocity and plastic strain rate have been used as nodal variables in 6 noded triangular elements. Finally a coupled FE equations are derived according to [1] and solved using a 2D finite element code.

[a]Corresponding author. E-mail: gsuman@iitk.ac.in.



(a) (b) (c)

Figure 1. (a) Variation of normalized hardness with depth of indentation normalized by l_*, (b) normalized load vs depth of indentation for sgp and conventional solid and (c) variation of $(H/H_o)^2$ with l_*/h.

RESULTS AND DISCUSSIONS

Figure 1 summarizes the results obtained from the finite element simulations. In figure 1(a) the normalized indentation hardness is plotted with the depth of indentation normalized by the l_* for conventional plasticity model and gradient plasticity model. The hardness in figure 1 has been normalized by the hardness from the conventional plasticity simulations. It is evident that as the indentation depth becomes less than $4l_*$, the hardness starts to increase rapidly exhibiting the indentation size effect. Figure 1(b) shows the load vs displacement curve for both conventional case and gradient plasticity (GP) simulation for indentation. At small depths the load at the indenter tip is large compared to that of the conventional, as the gradient of strain is high at small depths. Hardness rises corresponding to this behavior. Gradient of strain gradually disappears at higher depths and accordingly at higher depths load vs displacement for GP follow the conventional trend resulting in same hardness value at higher depths. Figure 1(c) shows the variation of square of normalized hardness H/H_o with inverse of depth of indentation. The results have been plotted for four categories of materials, classified according to their yield strength and the strain hardening parameter. As expected from FH model because of the inherent relation (3), none follow a linear trend as observed in the experiments. However if they are fitted with a straight line it is observed that the relative hardening is significantly low for high yield strength materials compared to that of the low yield strength materials. It can also be noticed that for the materials with higher strain hardening parameters the relative hardening at smaller depth is low compared to the materials with low strain hardening parameters. These trends also follow the experimental observations found at [4], e.g., the slope of the $(H/H_o)^2$ vs $1/h$ plot for MgO ($\sigma_Y = 2.82$ GPa) is significantly small compared to the same for annealed Cu ($\sigma_Y \approx 50$ Mpa). In addition to that, the characteristic length scale l_* also play a significant role in determining the behavior of the hardness variation. It is found that with increase in the l_*, the slope of the $(H/H_o)^2$ vs $1/h$ increases producing more relative hardening. This result also agrees well when compared with the data for annealed Cu ($l_* \approx 0.6$ μm) and Ir ($l_* \approx 5$ μm). Figure 1 also shows the effect of a hard substrate at the bottom of the solid of finite thickness (D) being indented. It is assumed that the dislocations can not move pass the boundary between the solid and the substrate. This is modeled in FH theory by employing the higher order boundary condition on plastic strain rate $\dot{\epsilon}^p$. The result shows that initially it follows the trend for general SG solid and approaches the bulk hardness as depth of indentation increases. However beyond $h/D > 0.2$, the hardness starts to rise again but with a lower slope compared to the same at low depths. The above behavior is also governed by the l_*. It can be mentioned here that Nix and Gao model is incapable of capturing this effect as no higher order boundary conditions exist in their model.

References

[1] U. Borg, C. F. Niordson, N. A. Fleck, and Viggo Tvergaard. A viscoplastic strain gradient analysis of materials with voids or inclusions. *International Journal of Solids and Structures*, 43:4906–4916, 2006.

[2] N. A. Fleck and J. W. Hutchinson. A reformulation of strain gradient plasticity. *Journal of the Mechanics and Physics of Solids*, 49:2245–2271, 2001. doi:10.1016/S0022-5096(01)00049-7.

[3] H. Gao, Y. Huang, W. D. Nix, and J. W. Hutchinson. Mechanism-based strain gradient plasticity - i. theory. *Journal of the Mechanics and Physics of Solids*, 47:1239–1263, 1999. doi:10.1016/S0022-5096(98)00103-3.

[4] Y. Huang, F. Zhang, K. C. Hwang, W. D. Nix, G. M. Pharr, and G. Feng. A model of size effects in nano-indentation. *Journal of Mechanics and Physics of Solids*, 54:1668–1686, 2006.

[5] Christian F. Niordson and Pia Redanz. Size-effects in plane strain sheet-necking. *Journal of the Mechanics and Physics of Solids*, 52:2431–2454, 2004. doi:10.1016/j.jmps.2004.05.009.

[6] J. G. Swadener, E. P. George, and G. M. Pharr. The correlation of the indentation size effects measured with indenters of various shapes. *Journal of Mechanics and Physics of Solids*, 50:681–694, 2002.