MODELING OF AISI L6 STEEL BEHAVIOUR IN THE PRESENCE OF THERMO-VISCOPLASTIC COUPLING

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Summary The paper investigates the influence of temperature rate terms on the constitutive response of AISI L6 steel subjected to nonisothermal conditions. The mathematical model is based on the extension of the 2 mechanisms – 1 yield criterion (2M1C) model [1], and the concept of thermodynamic modeling of coupled dissipative phenomena presented in [2].

INTRODUCTION

The increasing demands for high performance materials require the adequate constitutive modelling, as well as the appropriate predictions of the overall failure mechanisms under complex thermo-mechanical loads. When the elastic-(visco)plastic material is loaded so that not only inelastic strains develop, but also the temperature is changed, then thermo-elasticity and thermo-(visco)plasticity are encountered. The experimental results [3] proved that not only the temperature itself but also the heating rate makes a significant impact on parameters that determine carrying capacity at elevated temperatures. In [4] the discussion is made for the necessity of temperature rate terms in the context of hardening rules. More general case of the non-associated plasticity and non-associated damage is described in [2].
In the present analysis the 2 mechanisms – 1 yield criterion (2M1C) model identified in [1] to describe the elasto-viscoplastic behaviour of AISI L6 steel at different (but constant) temperatures, is extended to account for the effect of temperature change (nonisothermal conditions). A special attention is paid to the proper description of coupling between heating rate and plastic yielding.

EQUATIONS OF THERMO-ELASTIC-VISCOPLASTIC MATERIAL

State equations

The model is based on the assumption of small strains. Total strain is partitioned into an elastic, inelastic and thermal components, $\varepsilon_{ij} = \varepsilon_{ij}^E + \varepsilon_{ij}^I + \varepsilon_{ij}^\theta$, while the inelastic strain in 2M1C model can be partitioned itself in two different strain mechanisms, $\varepsilon_{ij}^I = A_1(\theta)\varepsilon_{ij}^{(1)} + A_2(\theta)\varepsilon_{ij}^{(2)}$. The complete set of state variables for the thermo-elastic-viscoplastic material consists of elastic strain ε_{ij}^E and absolute temperature θ; internal variables: kinematic and isotropic plastic hardening variables $\alpha_{ij}^{(1)}$, $\alpha_{ij}^{(2)}$ and $r^{(1)}$, $r^{(2)}$, respectively.
The state equations result from the assumed form of the state potential, which is here the Helmholtz free energy, decomposed into thermo-elastic ($\rho\psi^{te}$) and thermo-plastic ($\rho\psi^{tp}$) terms, after [1]:

$$\sigma_{ij} = \rho\frac{\partial\psi^{te}}{\partial\varepsilon_{ij}^E} = E_{ijkl}(\theta)(\varepsilon_{kl} - \varepsilon_{kl}^I) - \beta_{ij}(\theta)(\theta-\theta_0)\,,\quad \beta_{ij}(\theta) = E_{ijkl}(\theta)\alpha_{kl}^\theta(\theta)$$

$$X_{ij}^{(1)} = \rho\frac{\partial\psi^{tp}}{\partial\alpha_{ij}^{(1)}} = \frac{2}{3}(C_{11}(\theta)\alpha_{ij}^{(1)} + C_{12}(\theta)\alpha_{ij}^{(2)})\,,\quad X_{ij}^{(2)} = \rho\frac{\partial\psi^{tp}}{\partial\alpha_{ij}^{(2)}} = \frac{2}{3}(C_{22}(\theta)\alpha_{ij}^{(2)} + C_{12}(\theta)\alpha_{ij}^{(1)})$$

$$R^{(1)} = \rho\frac{\partial\psi^{tp}}{\partial r^{(1)}} = b_1(\theta)Q_1(\theta)r^{(1)}\,,\quad R^{(2)} = \rho\frac{\partial\psi^{tp}}{\partial r^{(2)}} = b_2(\theta)Q_2(\theta)r^{(2)}$$

In the above expressions $\alpha_{ij}^\theta(\theta)$ is the thermal expansion tensor, $C_{11}(\theta)$, $C_{22}(\theta)$, $C_{12}(\theta)$, $Q^{(1)}(\theta)$, $Q^{(2)}(\theta)$, $b_1(\theta)$, $b_2(\theta)$ are material parameters, which are temperature dependent (cf [1]).

Evolution equations

Potential of dissipation (F) is assumed not equal to plastic yield surface (non-associated thermo-viscoplasticity). This allows to obtain non-linear plastic hardening rules, which give more realistic description of the material response:

$$F = F^{vp}(f^p) + g(X_{ij}^{(k)})\,,\quad F^{vp} = \frac{K(\theta)}{n(\theta)+1}\left\langle\frac{f^p}{K(\theta)}\right\rangle^{n(\theta)+1}\,,\quad g = \sum_{k=1}^{2}\frac{M_k(\theta)}{m_k(\theta)+1}\left\langle\frac{J(X_{ij}^{(k)})}{M_k(\theta)}\right\rangle^{m_k(\theta)+1}$$

with $J(X_{ij}^{(k)}) = \sqrt{(3/2)X_{ij}^{'(k)}X_{ij}^{'(k)}}$, $K(\theta)$ and $n(\theta)$ are viscous coefficients, and $M_1(\theta)$, $M_2(\theta)$, $m_1(\theta)$, $m_2(\theta)$ are kinematic recovery terms, and $f^p = 0$ is the *J*-type plastic yield surface.

The rates of state variables are obtained by the use of the classical normality rule:

$$\dot{\varepsilon}_{kl}^{(i)} = \dot{\lambda}\frac{\partial f}{\partial \sigma_{kl}^{(i)}}, \quad \dot{\lambda} = \left\langle \frac{f}{K(\theta)} \right\rangle^{n(\theta)} = \sqrt{\frac{2}{3}(\dot{\varepsilon}_{ij}^{(1)}\dot{\varepsilon}_{ij}^{(1)} + \dot{\varepsilon}_{ij}^{(2)}\dot{\varepsilon}_{ij}^{(2)})}, \; i = 1,2$$

$$\dot{\alpha}_{kl}^{(i)} = \dot{\varepsilon}_{kl}^{(i)} - \frac{3}{2}\dot{\lambda}\frac{D_i(\theta)}{C_{ii}(\theta)}X_{kl}^{(i)} - \frac{3}{2}\frac{X_{kl}^{(i)}}{J(X_{kl}^{(i)})}\left\langle \frac{J(X_{kl}^{(i)})}{M_i(\theta)} \right\rangle^{m_i(\theta)}, \quad \dot{r}^{(i)} = \dot{\lambda}(1 - \frac{R^{(i)}}{Q_i(\theta)}), \; i = 1,2$$

The evolution equations for thermodynamic conjugate forces are derived taking the time rate of state equations, see Table 1.

Table 1. Kinetic equations for thermodynamic conjugate forces

	No coupling	Coupling with temperature
$\dot{\sigma}_{ij} =$	$E_{ijkl}\dot{\varepsilon}_{kl}^{E}$	$-[-\frac{\partial E_{ijkl}}{\partial \theta}(\varepsilon_{kl} - \varepsilon_{kl}^{I}) + \frac{\partial \beta_{ij}}{\partial \theta}(\theta - \theta_0) + \beta_{ij}]\dot{\theta}$
$\dot{X}_{kl}^{(i)} =$	$\frac{2}{3}(C_{ii}\dot{\alpha}_{kl}^{(i)} + C_{ij}\dot{\alpha}_{kl}^{(j)})$	$+\frac{2}{3}(\frac{\partial C_{ii}}{\partial \theta}\alpha_{kl}^{(i)} + \frac{\partial C_{ij}}{\partial \theta}\alpha_{kl}^{(j)})\dot{\theta}; i, j = 1,2; i \neq j, \text{no sum}$
$\dot{R}^{(i)} =$	$b_i(Q_i\dot{r}^{(i)} + \dot{Q}_i r^{(i)})$	$+(\frac{\partial b_i}{\partial \theta}Q_i + b_i\frac{\partial Q_i}{\partial \theta})r^{(i)}\dot{\theta}; i = 1,2, \text{no sum}$

By taking into account the temperature dependence of material characteristics the additional terms appear in the above evolution equations, which may play a significant role when solving high temperature problems, such as fire conditions or thermal shock problems.

Heat balance equation

In the case of thermo-elastic-viscoplastic material, the general coupled heat equation takes the following form:

$$\rho c_{\varepsilon}^{\theta}\dot{\theta} = -q_{i,i} + r - \theta P_{ij}(\dot{\varepsilon}_{ij} - \dot{\varepsilon}_{ij}^{I}) + \sigma_{ij}\dot{\varepsilon}_{ij}^{I} - (R^{(1)} - \theta\frac{\partial R^{(1)}}{\partial \theta})\dot{r}^{(1)} - (X_{ij}^{(1)} - \theta\frac{\partial X_{ij}^{(1)}}{\partial \theta})\dot{\alpha}_{ij}^{(1)} - (R^{(2)} - \theta\frac{\partial R^{(2)}}{\partial \theta})\dot{r}^{(2)} - (X_{ij}^{(2)} - \theta\frac{\partial X_{ij}^{(2)}}{\partial \theta})\dot{\alpha}_{ij}^{(2)}$$

which is nonlinear and fully coupled to mechanical problem. Heat flux is given by the Fourier's law: $q_i = -k_{ij}\theta_{,j}$.

NUMERICAL IMPLEMENTATION

The mathematic model is implemented into ABAQUS UMAT procedure and numerical simulations are performed to investigate the influence of thermo-viscoplastic coupling on the material response.The fully implicit backward Euler scheme is chosen, which is always stable and very accurate. Adopting the Newton-Raphson method, the iterative solution procedure is defined as $\Delta\mathbf{S}^{(k+1)} = \Delta\mathbf{S}^{(k)} - \left[\mathbf{J}^{(k)}\right]^{-1}\mathbf{R}^{(k)}$, where $\Delta\mathbf{S}$ is the vector containing the increments of the unknowns, $[\mathbf{J}] = \frac{\partial \mathbf{R}}{\partial \Delta\mathbf{S}}$ is the Jacobian matrix and $\mathbf{R}$ is a residual vector, containing the components $\mathbf{R}_{S_i} = \Delta S_i - \Delta\hat{S}_i$, where ΔS_i is a variable while $\Delta\hat{S}_i$ denotes the function resulting from the evolution rule for i-th variable S_i. The iteration procedure is stopped when the norm of $\mathbf{R}$ is sufficiently small. The state variables $\mathbf{S}_{n+1}^{k}$ in the current *k*-th iteration are expressed at the end of the step as the values at the beginning of the step $\mathbf{S}_n$ corrected by the current iterate increments $\Delta\mathbf{S}^k$: $\mathbf{S}_{n+1}^{k} = \mathbf{S}_n + \Delta\mathbf{S}^k$.

Acknowledgements

The financial support under grant number 2285/B/T02/2011/40 from the Polish Ministry of Science and Higher Education is gratefully acknowledged.

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