GENERAL CRITERIA OF MATERIAL INSTABILITIES

Wei Ma

Institute of Mechanics, Chinese Academy of Sciences, Beijing, 100190, China

In this work, we study the joint influences of material constitutive behaviors and loading conditions on the stability of plastic flow for thermal viscoplastic materials. Based on the linear perturbation analysis, a general characteristic spectral equation is deduced from general three-dimensional field equations and a universal criterion to estimate materials instability behaviors is proposed. The criterion shows that the instability takes place once thermal softening surpasses work hardening. Mathematically, it explicitly shows the effects of strain hardening, strain rate sensitivity, thermal softening and heat conduction as well as normal and shear stresses and strain rates on instability behaviors of materials. If specified loading conditions are assumed, corresponding criteria are derived from the criterion. The detail analysis and discussion on the influence of strain rate sensitivities and loading conditions on instability behaviors are performed.

INTRODUCTION

So far, material instability phenomena have widely received attention in academic community and many valuable criteria have been established for estimating material instability behaviors caused by various loadings [1-3]. However, it seems that a universal criterion comprehensively describing instability behaviors under general loading conditions is still lacking. This article will address some aspects of the problem on the bases of stability analysis of material deformation under general loading conditions.

INSTABILITY CRITERIA

Through linear stability analysis of the govern equations of a three-dimensional body of thermal viscoplastic material under general loadings, the general characteristic spectral equation is deduced as

$$-C_{13}C_{22}\alpha^3+\left[(C_{12}C_{24}-C_{13}C_{23})D_1k^2-C_{13}C_{22}C_{31}k^2-C_{13}C_{22}(D_2+D_3)\right]\alpha^2+\left[\begin{array}{l} C_{31}(C_{12}C_{24}-C_{13}C_{23})D_1k^2+(C_{12}C_{24}-C_{13}C_{23})+C_{21}(C_{12}D_4-C_{13}D_5)\\ +C_{12}(C_{21}D_6+C_{24}D_7+C_{24}D_8)-C_{13}(C_{21}D_9+C_{23}D_7+C_{23}D_8)\end{array}\right]k^2\alpha \quad (1)$$

$$+C_{31}(C_{12}C_{24}-C_{13}C_{23})k^4+\left[C_{12}(C_{21}D_{10}+C_{24}D_2+C_{24}D_3)-C_{13}(C_{21}D_{11}+C_{23}D_2+C_{23}D_3)\right]k^2=0$$

where C_{ij} are constants and D_k are dimensionless variables [4]. From the spectral equation, the universal criterion to evaluate instability behaviors is obtained as follows

$$\frac{9\Phi P_0}{5}\left[2(\eta_{12}\tau_{120}+\eta_{23}\tau_{230}+\eta_{31}\tau_{310})+(\eta_{11}\sigma_{110}+\eta_{22}\sigma_{220}+\eta_{33}\sigma_{330})+D_E\left(R_0+\frac{\lambda t_c}{c}\right)\right]-2\sqrt{\frac{3}{5}(\dot{\varepsilon}_{110}+\dot{\varepsilon}_{220}+\dot{\varepsilon}_{330}+\dot{\gamma}_{120}+\dot{\gamma}_{230}+\dot{\gamma}_{310})\rho\lambda\Phi P_0Q_0}>\rho cQ_0 \quad (2)$$

where $D_E=D_{ES}+D_{ET}$ has the dimension of strain rates and

$$D_{ES}=(2\eta_{12}-5/9)\dot{\gamma}_{120}+(2\eta_{23}-5/9)\dot{\gamma}_{230}+(2\eta_{31}-5/9)\dot{\gamma}_{310},\ D_{ET}=(\eta_{11}-5/9)\dot{\varepsilon}_{110}+(\eta_{22}-5/9)\dot{\varepsilon}_{220}+(\eta_{33}-5/9)\dot{\varepsilon}_{330} \quad (3)$$

The criterion (2) indicates that the material instability deals with complex thermodynamic process. Whether shear localization or necking instabilities closely depend on loading conditions. In physics, the criterion shows that the instability takes place once the thermal softening surpasses the work hardening. In mathematics, it explicitly shows the effects of strain hardening, strain rate sensitivity, thermal softening and heat conduction as well as normal and shear stresses and strain rates on instability behaviors. The terms inside the parentheses on the left-hand side of (2) are the dominant contribution and the terms within the radical sign are the secondary contribution for thermal softening, since, in many cases, the values of radical terms are about two orders less than the values of parentheses terms. Stress components always enhance the thermal softening and make plastic flow tend to instable; however, the effects of strain rate sensitivity and heat conduction on thermal softening depend on the proportion factors of strain components η_{ij}. The criterion may estimate the instability behaviors for rate-dependent, rate-independent, conductive and non-conductive materials, and describe the instability modes of shear localization and necking. Therefore, it is a universal criterion from which relevant criteria can be obtained for some specified loading cases. For example, in the plane loading case, we have

$$2\Phi P_0\left[2\tau_{ij0}+\sigma_{ii0}+\sigma_{jj0}-\left(\dot{\varepsilon}_{ii0}+\dot{\varepsilon}_{jj0}-\dot{\gamma}_{ij0}\right)\left(R_0+\frac{\lambda t_c}{c}\right)\right]-2\sqrt{6\rho\lambda\Phi P_0Q_0\left(\dot{\gamma}_{ij0}+\dot{\varepsilon}_{ii0}+\dot{\varepsilon}_{jj0}\right)}>3\rho cQ_0 \quad (4)$$

The criteria (4) describe the material instability behaviors occurring in x_1x_2, x_2x_3 and x_3x_1 coordinate planes when (i, j)=(1, 2); (2, 3) and (3, 1). In simple shear case we obtain

$$\Phi P_0\tau_{ij0}-2\sqrt{\rho\lambda\Phi P_0Q_0\dot{\gamma}_{ij0}}>\rho cQ_0 \quad (5)$$

When (i, j)=(1, 2); (2, 3) and (3, 1), the criteria (5) describe the shear localization instability occurring in the directions of x_1, x_2 and x_3 axes. The term containing D_Et_c in (2) denotes the plastic deformation of materials in the characteristic time scale of t_c. In the quasi-static loading cases, t_c is sufficiently large so that $E_E=D_Et_c$ denotes a finite plastic deformation. Moreover, heat conduction extends the thermal softening zone; therefore, the occurring probability of nonlocalization necking instability increases. For strain rate non-sensitive materials, the criterion (2) is changed into

$$\frac{9\Phi P_0}{5}\left[2\left(\eta_{12}\tau_{120}+\eta_{23}\tau_{230}+\eta_{31}\tau_{310}\right)+\left(\eta_{11}\sigma_{110}+\eta_{22}\sigma_{220}+\eta_{33}\sigma_{330}\right)+\frac{\lambda}{c}E_E\right]>\rho c Q_0 \tag{6}$$

The criterion (6) shows that the thermal softening in plastic deformation is caused by normal and shear stresses and heat conduction. The instable modes may be the shear localization instability or nonlocalization thermal softening instability. Therefore, what kinds of instability modes to happen depend on the amplitude proportion between shear and normal stresses and the natures of function E_E. In the cases of dynamic loading, the process of heat conduction is too late relative to material deformation, but strain rate is an important factor affecting instability behaviors; thus, the adiabatic shear band instability is possible. For the strain rate sensitive materials, the criterion (2) reduces into the form

$$\frac{9\Phi P_0}{5}\left[2\left(\eta_{12}\tau_{120}+\eta_{23}\tau_{230}+\eta_{31}\tau_{310}\right)+\left(\eta_{11}\sigma_{110}+\eta_{22}\sigma_{220}+\eta_{33}\sigma_{330}\right)+D_E R_0\right]>\rho c Q_0 \tag{7}$$

The criterion (7) describes the instability behaviors of materials under impacting loading conditions. At this time, the thermal softening is caused by normal and shear stresses and strain rate sensitivity.

Now, let us consider the influence of strain rate sensitivity R_0 related to different strain rates on instability behaviors. For simplicity, just consider the effects of normal strain rates with the second function D_{ET} in (3). By using the spatial plane equation $\eta_{11}+\eta_{22}+\eta_{33}=1$ and the condition $0<\eta_{11},\eta_{22},\eta_{33}<1$, we determine the response domain $\Delta Q_1Q_2Q_3$ of variables η_{11}, η_{22} and η_{33} in $\eta_{11}\eta_{22}\eta_{33}$-space (Fig. 1). Furthermore, the definition domain $\Delta P_1P_2P_3$ of argument of function D_{ET} in strain-space is obtained (Fig. 2). The domain $\Delta P_1P_2P_3$ is divided into four subdomains by three coordinate planes and locates below the origin of coordinate system. For any one of three triangle subdomains, we note that there is certainly a positive term and two negative terms on the right-hand side of function D_{ET} for rate-hardening materials ($R_0>0$). As an example, look at the function D_{ET} in the domain $\Delta P_1M_1M_2$. The three vertex coordinates of $\Delta P_1M_1M_2$ are found to be $P_1(4/9, -5/9, -5/9)$, $M_1(0, -1/9, -5/9)$ and $M_2(0, -5/9, -1/9)$ from the conditions $5/9<\eta_{11}<1$, $0<\eta_{22}<4/9$ and $0<\eta_{33}<4/9$. Thus, there must be $\eta_{11}-5/9>0$, $\eta_{22}-5/9<0$ and $\eta_{33}-5/9<0$ in $\Delta P_1M_1M_2$ for the three coefficients of the right-hand side terms of function D_{ET}. The strain rate sensitivity $D_{ET}R_0$ corresponding to the positive $(\eta_{11}-5/9)\,\dot{\varepsilon}_{110}$ reinforces the thermal softening and is helpful for instability occurrence; but the rate sensitivities to other two negative $(\eta_{22}-5/9)\,\dot{\varepsilon}_{220}$ and $(\eta_{33}-5/9)\,\dot{\varepsilon}_{330}$ always reduce the thermal softening and counterwork instability occurrence. The net effect of strain rate sensitivity $D_{ET}R_0$ on instability behaviors depend on the competition of the two kinds of mechanisms of rate-sensitivity and the relative magnitudes of strain rates. For the central hexagonal domain, it can be found that the three terms of function D_{ET} are constantly negative; thus, the strain rate sensitivity invariably reduces thermal softening and leads to stable plastic flow. By the same way, we can study the effects of strain rate sensitivity $D_{ES}R_0$ related with shear strain rates on instability behaviors and further investigate the effects of heat conduction on instability behaviors based on first function D_{ES} in (3).

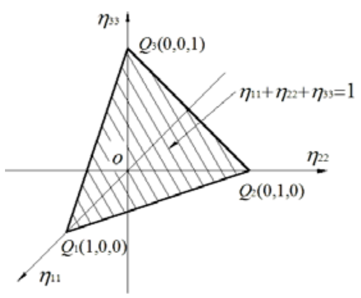

Fig.1. The response domain $\Delta Q_1Q_2Q_3$ of variables η_{11}, η_{22} and η_{33}.

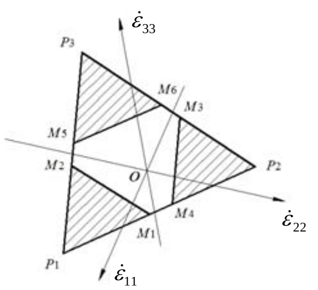

Fig.2. The definition domain $\Delta P_1P_2P_3$ of argument of function D_{ET}

CONCLUSIONS

A universal criterion is obtained to estimate materials instability behaviors. The criterion shows that plastic flow loses its stability once the thermal softening surpasses the work hardening. Moreover, it explicitly shows the effects of strain hardening, strain rate sensitivity, thermal softening and heat conduction as well as normal and shear stresses and strain rates on instability behaviors of materials. It also reveals that the effects of strain rate sensitivity and heart conduction on thermal softening are related with various strain rates. For the cases of plane strain, simple shear, quasi-static and dynamic loadings, corresponding criteria are obtained from the criterion. Therefore, the instability criterion gives the universal conditions to estimate material instability.

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